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NOUVELLE SÉRIE

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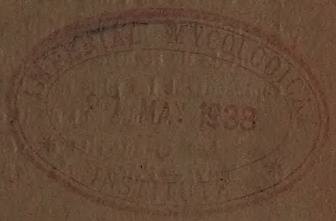

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MATHÉMATIQUES

SUR LA MEILLEURE APPROXIMATION DE $(x-c)^p$

Par SERGE BERNSTEIN, de l'Académie

Dans un travail qui paraîtra prochainement, j'ai démontré que $p>0$ étant un nombre donné, la meilleure approximation $E_n|x|^p$ sur le segment $(-1,+1)$ satisfait à l'égalité asymptotique

$$E_n|x|^p \sim \frac{1}{\pi}\left|\sin\frac{\pi}{2}p\right|\frac{L(p)}{n^p}=\frac{\mu(p)}{n^p}, \qquad (1)$$

où $L(p)$ est une fonction continue de p.

Je me propose de montrer que l'on a pour toute valeur donnée de $c\,(-1<c<1)$:

$$\lim_{n=\infty} n^p E_n|x-c|^p=(1-c^2)^{\frac{p}{2}}\mu(p). \qquad (2)$$

Dans ce but nous démontrerons les propositions suivantes:

Théorème I. *En désignant* * *par* $E_{n,\,q(x)}[f(x);\,(a,b)]$ *la meilleure approximation avec le poids* $q(x)$ *par des polynômes de degré* n de $f(x)$ *sur le segment* (a,b), on a

$$E_{n,\,n^{x^2}}[|x|^p;\,(a,b)]<E_n(|x|^p;\,(a,b))\,(1+\alpha_n),\quad {}_{(-1\leqslant a<0<b\leqslant 1)} \qquad (3)$$

où $\alpha_n\to 0$ pour $n\to\infty$.

En effet, posons $h=\log n$ et

$$n^{x^2}=e^{hx^2}=1+hx^2+\ldots+\frac{h^kx^{2k}}{k!}+\varepsilon^h=R_{2k}(x)+\varepsilon_k,$$

$$n^{-x^2}=e^{-hx^2}=1-hx^2+\ldots+(-1)^k\frac{h^kx^{2k}}{k!}+\varepsilon'_k=Q_{2k}(x)+\varepsilon'_k$$

où $k=[\log^2 n]$; donc, $|\varepsilon_k|<\left(\frac{he}{k+1}\right)^{k+1}$, $|\varepsilon'_k|<\left(\frac{he}{k+1}\right)^{k+1}$ pour $|x|\leqslant 1$ tendent vers zéro avec $\frac{1}{n}$. On a aussi, évidemment,

$$|R_{2k}(x)Q_{2k}(x)-1|<2e^h\left(\frac{he}{k+1}\right)^{k+1}<2n\left(\frac{e}{\log n}\right)^{\log^2 n}=\beta_n\to 0.$$

---

* Pour abréger, nous écrirons $E_{n,\,q(x)}(f(x)=E_{n,\,q(x)}[f(x);\,(-1,+1)]$, $E_n f(x);\,(a,b))=E_{n,\,1}[f(x);\,(a,b)]$.

Donc,

$$E_n(|x|^p;\ (a,b))(1+\beta_n) > E_{n,\,R_{2k}(x)\,Q_{2k}(x)}(|x|^p,(a,b)) > \\ > E_{n+2k,\,R_{2k}(x)}(|x|^p Q_{2k}(x);(a,b)) > \\ > E_{n+2k,\,R_{2k}(x)}[|x|^p;(a,b)] - \sum_{i=1}^{k} \frac{h^i}{i!} E_{n+2k,\,R_{2k}(x)}(|x|^{p+2i};(a,b)).$$

En tenant compte de l'inégalité connue (1)

$$m^p E_m|x|^p < \frac{4}{\pi}\left|\sin\frac{\pi p}{2}\right|\Gamma(p)$$

et du fait que $|R_{2k}(x)| \leqslant n$ pour $|x| \leqslant 1$, nous avons:

$$m^{p+2i-1} E_{m,\,R_{2k}(x)}[|x|^{p+2i};(a,b)] < \frac{4}{\pi}\Gamma(p+2i).$$

Donc,

$$E_{n+2k,\,R_{2k}(x)}(|x|^p;(a,b)) < (1+\beta_n) E_n(|x|^p;(a,b)) + \\ + \frac{4}{\pi}\sum_{i=1}^{k} \frac{h^i \Gamma(p+2i)}{i!\, n^{p+2i-1}} < (1+\gamma_n) E_n(|x|^p;(a,b)), \qquad (4)$$

où $\gamma_n \to 0$ (pourvu que $\frac{k}{n} \to 0$).

D'autre part, on voit facilement que l'on a pour toute valeur de $\delta_m > 0$:

$$E_{m-1}(|x|^p;(a,b)) < E_m[|x|^p;(a,b)] \cdot \frac{(1+\delta_m)^m + 1}{(1+\delta_m)^{m-p} - 1},$$

d'où on tire, en posant $(1+\delta_m)^{m-p} = 2m$, que

$$E_n[|x|^p;(a,b)] < \frac{2(n+k)+1}{2n+1} \prod_{i=1}^{2k}\left(1 + \frac{2p\log 2(n+i)}{n+i-p}\right) E_{n+2k}(|x|^p;(a,b)) < \\ < (1+\varepsilon_n) E_{n+2k}(|x|^p;(a,b)) \qquad (5)$$

où $\varepsilon_n \to 0$, pourvu que $\frac{k\log n}{n} \to 0$; il suffit pour achever la démonstration de multiplier les inégalités (4) et (5).

Corollaire I. *Quels que petits que soient les nombres* $\alpha > 0$, $\beta > 0$, *il est possible de fixer un nombre* n_0 *tel que pour toute valeur entière de* $n > n_0$ *il existe des polynômes* $P_n(x)$ *de degré* $n > n_0$ *satisfaisant à l'inégalié:*

$$|P_n(x) - |x-c|^p| < (1+\beta) E_n|x-c|^p, \qquad (6)$$

sur tout le segment $(-1,+1)$ *et à l'inégalité plus forte:*

$$|P_n(x) - |x-c|^p| < \beta E_n|x-c|^p, \qquad (6\text{ bis})$$

dans les intervalles $(-1, c-\alpha)$, $(c+\alpha, 1)$.

En effet, en vertu du théorème 1, il existe un polynôme $S_n(x)$ de degré n satisfaisant à l'inégalité:

$$||x|^p - S_n(x)| < E_n(|x|^p;(-1,b))\ (1+\alpha_n),$$

sur le segment $(-1, b)$ et à l'inégalité plus forte:

$$||x|^p - S_n(x)| < \frac{1+\alpha_n}{n^{\delta^2}} E_n(|x|^p;(-1,b)),$$

dans les intervalles $(-1,-\delta)$ et (δ,b), où $b=\frac{1-c}{1+c}$, $\delta=\frac{\alpha}{1+c}$. Soit $c>0$, pour fixer les idées.

Donc, après le changement $x=\frac{x_1-c}{1+c}$, le polynôme $P_n(x_1)=$ $=(1+c)^p S_n\left(\frac{x_1-c}{1+c}\right)$ satisfait à l'inégalité:

$$||x_1-c|^p-P_n(x_1)|<(1+\alpha_n)E_n|x_1-c|^p,$$

sur tout le segment $(-1,+1)$, et à l'inégalité:

$$||x_1-c|^p-P_n(x_1)<\frac{1+\alpha_n}{n^{\delta^2}}E_n|x_1-c|^p,$$

dans les intervalles $(-1,c-\alpha)$, $(c+\alpha,1)$; ainsi, pour réaliser les inégalités (6) et (6 bis) il suffit de faire n_0 assez grand pour avoir $\alpha_{n_0}\leqslant\beta$ et $1+\beta<\beta n_0^{\left(\frac{\alpha}{1+c}\right)^2}$.

2. Désignons par $E_n[f(x);(a,b);L]$, où $-1<a<b<1$, la meilleure approximation de $f(x)$ sur le segment (a,b) par des polynômes $P_n^*(x)$ de degré n satisfaisant à la condition $|P_n^*(x)|\leqslant L$ sur tout le segment $(-1,+1)$.

Théorème II. *Quels que soient les nombres donnés $p>0$, $h>0$, $-1<c<1$, et quel que petit que soit le nombre $\varepsilon>0$, on a pour n suffisamment grand l'inégalité:*

$$E_n(|x-c|^p;(a,b);n^h>(1-\varepsilon)E_n|x-c|^p \tag{7}$$

pourvu que $c-a\geqslant\frac{1}{n^\alpha}$, $b-c\geqslant\frac{1}{n^\alpha}$ avec $0<\alpha<\frac{1}{2}$.

En effet, posons $m=n+2l$, où le nombre entier $l=n^\rho$, $(2\alpha<\rho<1)$, $q(x)=\left[1-\left(\frac{x-c}{1+|c|}\right)^2\right]^l$. Donc,

$$E_{n,q(x)}|x-c|^p\geqslant E_m(q(x)|x-c|^p>E_m|x-c|^p-$$

$$-\sum_1^l C_l^i E_n\left(\frac{|x-c|^{p+2i}}{(1+|c|)^{2i}}\right).$$

On sait (1), qu'il existe une constante λ_p telle que

$$\frac{E_m|x|^{p+2i}}{E_m|x|^p}<\lambda_p\frac{p(p+1)\ldots(p+2i-1)}{m^{2i}};$$

donc,

$$E_{n,q(x)}|x-c|^p>E_m|x-c|^p\left\{1-\frac{\lambda_p}{(1-|c|)^p}\sum_1^l C_l^i\frac{p(p+1)\ldots.(p+2i-1)}{m^{2i}}\right\}>$$

$$>(1-\varepsilon_n)E_n|x-c|^p, \tag{8}$$

où $\varepsilon_n\to 0$, pour $n\to\infty$. Il résulte de plus de (8) qu'il existe une constante A_p, telle que

$$E_{n,q(x)}|x-c|^p>\frac{A_p}{n^p}. \tag{9}$$

Admettons à présent que l'on a $|P_n^*(x)| \leqslant n^h$ sur $(-1, +1)$; nous aurons donc pour $-1 \leqslant x \leqslant c - \frac{1}{n^\alpha}$ et $c + \frac{1}{n^\alpha} \leqslant x \leqslant 1$,

$$q(x)|P_n^*(x) - |x-c|^p| < \left[1 - \frac{1}{(1+|c|)^2 n^{2\alpha}}\right]^l (n^h + 2^p) <$$

$$< \frac{1}{n^{p+1}} < E_{n,\,q(x)}|x-c|^p.$$

Par conséquent, si l'affirmation du théorème II était inexacte, l'inégalité (8) serait impossible.

Corollaire II. *Soit*

$$f(x) = \sum_{i=1}^{h} A_i|x-a_i|^p + \varphi(x), \quad (-1 < a_i < 1) \tag{10}$$

où $p > 0$ n'est pas un entier pair, la fonction $\varphi(x)$ possédant une dérivée bornée d'ordre $[p+1]$; soit M_n la plus grande des valeurs $E_n A_i|x-a_i|^p$. Dans ces conditions quel que petit que soit $\beta > 0$ il est possible de fixer un nombre n_0 tel que pour toute valeur entière $n > n_0$, on a:

$$1 - \beta < \frac{E_n(f(x))}{M_n} < 1 + \beta. \tag{11}$$

En effet, à cause du corollaire I, nous avons

$$E_n(f(x)) < (1+\beta)M_n.$$

D'autre part, s'il existait des polynômes $P_n(x)$ de degrés n indéfiniment croissants pour lesquels $|P_n(x) - f(x)| \leqslant M_n(1-\beta)$, on tomberait en contradiction avec le théorème II, car d'après le corollaire I il existe des polynômes $Q_{n,i}(x)$ de degrés n approchant $f(x) - A_i|x-a_i|^p$ avec une erreur moindre que $\frac{1}{2}\beta M_n$ sur $(a_i - \alpha, a_i + \alpha)$; il existerait donc pour chaque i un polynôme $R_{n,i}(x) = P_n(x) - Q_{n,i}(x)$ borné sur le segment $(-1, +1)$ et tel que

$$|R_{n,i}(x) - A_i|x-a_i|^p| = |P_n(x) - f(x) + (f(x) - A_i|x-a_i|^p - Q_{n,i}(x))| <$$

$$< M_n\left(1 - \frac{1}{2}\beta\right)$$

sur $(a_i - \alpha, a_i + \alpha)$.

3. Soit, en particulier, $(-1 < a < 1)$

$$f(x) = |x^2 - a^2|^p = |x-a|^p|x+a|^p = |2a|^p[|x-a|^p + |x+a|^p] + \varphi(x),$$

où

$$\varphi(x) = [|x-a|^p - |2a|^p][|x+a|^p - |2a|^p] - |2a|^{2p}$$

satisfait évidemment à la condition du corollaire II. Donc, on a pour n assez grand (quel que petit que soit $\alpha > 0$)

$$1 - \alpha < \frac{E_n|x^2-a^2|^p}{|2a|^p E_n|x-a|^p} < 1 + \alpha. \tag{12}$$

Admettons à présent que la formule limite

$$\lim_{n=\infty} n^p E_n|x-a|^p = H(a) \tag{13}$$

soit établie pour une valeur de a (il en est ainsi pour $a=0$). On doit évidemment avoir $H(a)=H(-a)$. Posons $a=2a_1^2-1$, $x=2z-1$; nous aurons

$$E_n\left(\left|z-\frac{a+1}{2}\right|^p;(0,1)\right)=E_n\left(\left|\frac{x-a}{2}\right|^p;\ (-1,+1)\right);$$

donc, d'après (13),

$$\lim_{n=\infty}(2n)^p E_n(|z-a_1^2|^p;(0,1))=H(a),$$

et à cause de

$$E_n(|z-a_1^2|^p;\ (0,1))=E_{2n}(|x^2-a_1^2|^p;\ (-1,1))=$$
$$=E_{2n+1}(|x^2-a_1^2|^p;\ (-1,+1)),$$

on a:

$$\lim_{n=\infty} n^p E_n|x^2-a_1^2|^p=H(a).$$

Donc, pour toute valeur de $n\to\infty$, on a, d'après (12),

$$\lim_{n=\infty} n^p E_n|x-a_1|^p=\frac{H(a)}{|2a_1|^p}=H(a_1), \tag{14}$$

de sorte que l'égalité limite (13) se trouve également établie pour $a_1=\pm\sqrt{\frac{1+a}{2}}$. D'une façon analogue, nous établissons successivement les égalités limites, où $a_{k+1}=\pm\sqrt{\frac{1+a_k}{2}}$,

$$\lim_{n=\infty} n^p E_n|x-a_{k+1}|^p=\frac{H(a_k)}{|2a_{k+1}|^p}=H(a_{k+1}). \tag{14 bis}$$

Posons $a=\cos\varphi$, $a_k=\cos\varphi_k$, $H(a)=H(\cos\varphi)=F(\varphi)$; donc, pour $a=0$, $\varphi=\frac{\pi}{2}$, et on obtient successivement $\varphi_k=\frac{1}{2}\varphi_{k-1}=\frac{l\pi}{2^{k+1}}$, où l est un nombre impair quelconque. Ainsi la formule limite (13) est établie pour $a_k=\cos\varphi_k$, où l'angle $\varphi_k=\frac{l\pi}{2^{k+1}}$. Pour déterminer $H(a_k)=F(\varphi_k)$ nous avons l'équation:

$$\frac{F(\varphi_{k-1})}{|2\cos\varphi_k|^p}=\frac{F(2\varphi_k)}{|2\cos\varphi_k|^p}=F(\varphi_k), \tag{15}$$

où $F\left(\frac{\pi}{2}\right)=H(0)=\mu(p)$. Donc, $F(\varphi_1)=F\left(\frac{\pi}{4}\right)=\mu(p)(\sin\varphi_1)^p$; et, d'après (15), si $F(\varphi_{k-1})=\mu(p)|\sin\varphi_{k-1}|^p$, on a aussi:

$$F(\varphi_k)=\mu(p)\left|\frac{\sin 2\varphi_k}{2\cos\varphi_k}\right|^p=\mu(p)|\sin\varphi_k|^p.$$

Ainsi l'égalité:

$$\lim_{n=\infty} n^p|x-a|^p=\mu(p)(1-a^2)^{\frac{p}{2}} \tag{16}$$

est vraie pour toutes les valeurs $a=\cos\frac{l\pi}{2^k}$ (pour toutes les valeurs entières de k et $l<2^k$).

Mais quel que soit c $(0<c<1)$, en posant

$$0<u_0=\cos\frac{l\pi}{2^k}\leqslant c<\cos\frac{l-1}{2^k}\pi=u_1,$$

on a:

$$\lim_{n=\infty} n^p E_n\left(|x-c|^p; \left(-\frac{c}{u_1}, \frac{c}{u_1}\right)\right) =$$

$$= \lim_{n=\infty} n^p \left(\frac{c}{u_1}\right)^p E_n|x-u_1|^p = \left(\frac{c}{u_1}\right)^p \mu(p)(1-u_1^2)^{\frac{p}{2}},$$

$$\lim_{n=\infty} n^p E_n\left(|x-c|^p; \left(-\frac{c}{u_0}, \frac{c}{u_0}\right)\right) = \left(\frac{c}{u_0}\right)^p \mu(p)(1-u_0^2)^{\frac{p}{2}}.$$

Donc, à cause des inégalités évidentes

$$n^p E_n\left(|x-c|^p; \left(-\frac{c}{u_1}, \frac{c}{u_1}\right)\right) \leqslant n^p E_n|x-c|^p \leqslant$$

$$\leqslant n^p E_n\left(|x-c|^p; \left(-\frac{c}{u_0}, \frac{c}{u_0}\right)\right),$$

l'égalité (2) est exacte pour toute valeur de c $(-1 < c < 1)$.

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LITTÉRATURE CITÉE

[1] С. Н. Бернштейн, Экстремальные свойства полиномов, гл. II, § 6.

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MATHÉMATIQUES

LIMITATION DES MODULES DES DÉRIVÉES SUCCESSIVES DES SOLUTIONS DES ÉQUATIONS DU TYPE PARABOLIQUE

SERGE BERNSTEIN, de l'Académie

1. Dans un travail consacré aux équations différentielles stochastiques qui paraîtra prochainement j'ai indiqué une méthode pour établir l'existence et calculer approximativement la solution de l'équation parabolique:

$$\frac{\partial z}{\partial \lambda} = Az + \sum_{i=1}^{b} a_i z_i' + \sum_{\substack{i=1 \\ k=1}}^{b} B_{ik} z_{ik}'' \quad \left(z_i' = \frac{\partial z}{\partial x_i},\ z_{ik}'' = \frac{\partial^2 z}{\partial x_i \partial x_k} \right) \tag{1}$$

$\left(\text{où } \sum B_{ik} u_i u_k \text{ est une forme quadratique positive définie} \right)$ lorsque la valeur de $z(\lambda, x_1, x_2, \ldots x_h)$ est donnée pour $\lambda = 0$. Il fallait seulement supposer que cette valeur initiale $z(0, x_1, \ldots, x_h) = \varphi(x_1, \ldots, x_h)$ possède des dérivées des trois premiers ordres par rapport à x_i. Cependant, il est facile de montrer que la méthode serait applicable sous la seule condition que la fonction $\varphi(x_1, \ldots, x_h)$ soit bornée et intégrable, pourvu qu'on puisse donner une limite supérieure des modules des dérivées partielles des deux premiers ordres $|z_i'|, |z_{ik}''|$ dans toute région donnée $S\left(0 < t_0 \leqslant \lambda \leqslant T, \sum_{i=1}^{h} x_i^2 \leqslant R^2 \right)$ dépendant seulement du maximum du module de la fonction initiale $\varphi(x_1, \ldots, x_h)$ en admettant l'existence des dérivées partielles de z jusqu'au quatrième ordre.

Il importe donc de démontrer qu'une telle limitation est effectivement possible. L'idée du procédé qui nous permettra d'atteindre ce but est analogue à celle que j'ai utilisée en 1910 dans le cas des équations elliptiques.

Pour plus de netteté je considérerai d'abord le cas de $h = 1$, $A = 0$

$$\frac{\partial z}{\partial \lambda} = a(x, \lambda) \frac{\partial z}{\partial x} + B(x, \lambda) \frac{\partial^2 z}{\partial x^2} \quad (B > 0)$$

à laquelle satisfait la fonction intégrale des probabilités $z(x, \lambda)$ d'une seule variable aléatoire x pour toute valeur de $\lambda \geqslant 0$, égale à $z(x, 0) = \varphi(x)$ pour $\lambda = 0$. On sait que $0 \leqslant z \leqslant 1$.

Posons

$$u=\lambda(R^2-x^2)^2\left(\frac{\partial z}{\partial x}\right)^2+cz^2,$$

où $R>0$ est un nombre donné arbitrairement grand et $c>0$ est une constante que nous fixerons plus tard. Nous considérons la fonction u dans une région S, où $0\leqslant\lambda\leqslant T$, $|x|\leqslant R$. Il est clair que, si u dépasse c, il atteint son maximum soit à l'intérieur de S, soit pour $\lambda=T$, de sorte qu'on a nécessairement en ce point

$$\frac{\partial^2 u}{\partial x^2}\leqslant 0,\quad \frac{\partial u}{\partial\lambda}\geqslant 0,$$

or

$$\frac{\partial^2 u}{\partial x^2}=2\lambda\left\{R^2-x^2)^2\left[\left(\frac{\partial^2 z}{\partial x^2}\right)^2+\frac{\partial z}{\partial x}\frac{\partial^3 z}{\partial x^3}\right]-8x(R^2-x^2)\frac{\partial z}{\partial x}\frac{\partial^2 z}{\partial x^2}+\right.$$
$$\left.+(6x^2-2R^2)\left(\frac{\partial z}{\partial x}\right)^2\right\}+2c\left[\left(\frac{\partial z}{\partial x}\right)^2+z\frac{\partial^2 z}{\partial x^2}\right]\leqslant 0,$$
$$\frac{\partial u}{\partial\lambda}=(R^2-x^2)^2\left(\frac{\partial z}{\partial x}\right)^2+2\lambda(R^2-x^2)\frac{\partial z}{\partial x}\left[a_x^1\frac{\partial z}{\partial x}+(a+B_x')\frac{\partial^2 z}{\partial x^2}+B\frac{\partial^3 z}{\partial x^3}\right]+$$
$$+2cz\left[a\frac{\partial z}{\partial x}+B\frac{\partial^2 z}{\partial x^2}\right]\geqslant 0.$$

Donc

$$B\frac{\partial^2 u}{\partial x^2}-\frac{\partial u}{\partial\lambda}=2\lambda B(R^2-x^2)^2\left(\frac{\partial^2 z}{\partial x^2}\right)^2+\left(\frac{\partial z}{\partial x}\right)^2[2c-(R^2-x^2)^2-$$
$$-2\lambda a_x^1(R^2-x^2)+4\lambda B(3x^2-R^2)]-$$
$$-2\lambda(R^2-x^2)(8Bx+a+B_x')\frac{\partial z}{\partial x}\frac{\partial^2 z}{\partial x^2}-2caz\frac{\partial z}{\partial x}\leqslant 0,$$

et a fortiori

$$\left(\frac{\partial z}{\partial x}\right)^2\left[2c-(R^2-x^2)^2-2\lambda a_x'(R^2-x^2)+\right.$$
$$\left.+4\lambda B(3x^2-R^2)-\lambda\frac{(8Bx+a+B_x')^2}{2B}\right]\leqslant 2caz\frac{\partial z}{\partial x}.$$

Par conséquent, en supposant c assez grand pour que le coefficient de $\left(\frac{\partial z}{\partial x}\right)^2$ soit supérieur à c dans la région S, nous voyons que $\left|\frac{\partial z}{\partial x}\right|<2L$, si on a $|a|\leqslant L$ dans cette région. Donc

$$u<4TR^4L^2+c=M^2$$

et, par conséquent, on a dans toute la région S:

$$\left|\frac{\partial z}{\partial x}\right|<\frac{M}{(R^2-x^2)\sqrt{\lambda}}. \qquad (2)$$

La limitation des dérivées suivantes se fait d'une façon analogue; je ne m'y arrêterai pas, car cette extension ne présente aucune difficulté dans le cas d'une seule variable x, pour passer au cas de plusieurs variables, où le passage aux dérivées suivantes présente quelques complications techniques.

2. Il suffira d'examiner l'équation:

$$\frac{\partial z}{\partial\lambda}=Az+ap+bq+r+t\quad\left(p=\frac{\partial z}{\partial x},\ q=\frac{\partial z}{\partial y},\ r=\frac{\partial^2 z}{\partial x^2},\ s=\frac{\partial^2 z}{\partial x\,\partial y},\ t=\frac{\partial^2 z}{\partial y^2}\right) \qquad (3)$$

à laquelle est réductible l'équation parabolique (1) pour $h=2$.

Soit S une région, où $0\leqslant\lambda\leqslant T$, $x^2+y^2\leqslant R^2$; par hypothèse $|Z|\leqslant N$ est borné dans cette région.

Posons

$$u = \lambda F(p^2+q^2)+cz^2,$$

où $F=(R^2-x^2-y^2)^2$. Remarquons, comme plus haut, que si u dépasse c, il atteint son maximum en un point de S, où

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\leqslant 0,\quad \frac{\partial u}{\partial \lambda}\geqslant 0;$$

de plus, puisque p^2+q^2 est invariant par rapport à la rotation des axes, nous pouvons, pour simplifier un peu les formules, poser $q=0$ en ce point.

Or

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=\lambda\left\{F(2r^2+4s^2+2t^2)+4p(rF'_x+sF'_y)+\right.$$
$$\left.+p^2(F''_{x^2}+F''_{y^2})+2Fp\left(\frac{\partial r}{\partial x}+\frac{\partial t}{\partial x}\right)\right\}+2cz(r+t)+2cp^2,$$
$$\frac{\partial u}{\partial \lambda}=Fp^2+2\lambda Fp\frac{\partial p}{\partial \lambda}+2cz\frac{\partial z}{\partial \lambda}=Fp^2+2\lambda Fp\left[A'_x z+Ap+\right.$$
$$\left.+a'_x p+ar+bs+\frac{\partial r}{\partial x}+\frac{\partial t}{\partial x}\right]+2cz(Az+ap+r+t),$$

d'où

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}-\frac{\partial u}{\partial \lambda}=\lambda\left\{2F[r^2+2s^2+t^2-p(A'_x z+Ap+a'_x p+ar+bs)]+\right.$$
$$\left.+4p[rF'_x+sF'_y]+p^2[F''_{x^2}+F''_{y^2}]\right\}+(2c-F)p^2-2cz(Az+ap)\leqslant 0.$$

De plus, en remplaçant l'ensemble des termes contenant r, s, t par leur minimum, nous obtenons a fortiori

$$p^2\left[2c-F+4(F''_{x^2}+F''_{y^2})-2\lambda F(A+a'_x)-\frac{2(\lambda Fa+2F'_x)^2+(\lambda Fb+2F'_y)^2}{4\lambda F}\right]-$$
$$-2\lambda FA'_x pz-2cz(Az+ap)\leqslant 0.$$

Grâce au fait que $\frac{F'^2_x+F'^2_y}{F}=16(x^2+y^2)\leqslant 16R^2$ reste borné, nous pouvons choisir la constante c assez grande pour que le coefficient de p^2 devienne supérieur à c; ainsi, p devra satisfaire à une inégalité de second degré

$$p^2+h_1 p+h_2\leqslant 0$$

à coefficients h_1 et h_2 bornés. Donc, au point, où u est maximum, p^2 ne peut dépasser une constante fixe; d'où nous concluons, comme plus haut, qu'on doit avoir dans toute la région S

$$p^2+q^2<\frac{M_1^2}{\lambda(R^2-x^2-y^2)^2} \qquad (4)$$

avec une même valeur de la constante M_1.

3. Du moment qu' on a trouvé une limite supérieure $\frac{M_1}{(R^2-x^2-y^2)\sqrt{\lambda}}$ de $\sqrt{p^2+q^2}$ pour tous les points de la région S, considérons une région S_1, où $\lambda_0\leqslant\lambda\leqslant T$, $x^2+y^2<R_1^2$ en supposant $\lambda_0>0$, $R_1<R$, et envisageons la fonction

$$u=\lambda_1 F(r^2+2s^2+t^2)+c(p^2+q^2)$$

où

$$\lambda_1=\lambda-\lambda_0,\ F=(R_1^2-x^2-y^2)^2.$$

Remarquons aussi que $r^2+2s^2+t^2$ est invariant par rapport à une rotation des axes, de sorte que nous pouvons pour simplifier un peu les calculs supposer $s=0$ au point, où u atteint son maximum: en ce point on a aussi, comme précédemment,

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\leqslant 0,\quad \frac{\partial u}{\partial \lambda}\geqslant 0.$$

Or,

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=2\lambda F\left[r\left(\frac{\partial^2 r}{\partial x^2}+\frac{\partial^2 r}{\partial y^2}\right)+\right.$$
$$+t\left(\frac{\partial^2 t}{\partial x^2}+\frac{\partial^2 t}{\partial y^2}\right)+\left(\frac{\partial r}{\partial x}\right)^2+3\left(\frac{\partial r}{\partial y}\right)^2+3\left(\frac{\partial t}{\partial x}\right)^2+\left.\left(\frac{\partial t}{\partial y}\right)^2\right]+$$
$$+4\lambda F'_x\left(r\frac{\partial r}{\partial x}+t\frac{\partial t}{\partial x}\right)+4\lambda F'_y\left(r\frac{\partial r}{\partial y}+t\frac{\partial t}{\partial y}\right)+\lambda\left(F''_{x^2}+F''_{y^2}\right)\left(r^2+t^2\right)+$$
$$+2c\left[p\left(\frac{\partial r}{\partial x}+\frac{\partial t}{\partial x}\right)+q\left(\frac{\partial r}{\partial y}+\frac{\partial t}{\partial y}\right)+r^2+t^2\right],$$
$$\frac{\partial u}{\partial \lambda}=F\left(r^2+t^2\right)+2\lambda F\left(r\frac{\partial r}{\partial \lambda}+t\frac{\partial t}{\partial \lambda}\right)+2c\left(p\frac{\partial p}{\partial \lambda}+q\frac{\partial q}{\partial \lambda}\right)=$$
$$=F\left(r^2+t^2\right)+2\lambda F\left\{r\left[\frac{\partial^2 r}{\partial x^2}+\frac{\partial^2 t}{\partial x^2}+a\frac{\partial r}{\partial x}+b\frac{\partial r}{\partial y}+\left(2a'_x+A\right)r+\right.\right.$$
$$\left.+\left(a''_{x^2}+2A'_x\right)p+b''_{x^2}q+A''_{x^2}z\right]+$$
$$\left.+t\left[\frac{\partial^2 r}{\partial y^2}+\frac{\partial^2 t}{\partial y^2}+a\frac{\partial t}{\partial x}+b\frac{\partial t}{\partial y}+\left(2b'_y+A\right)t+a''_{y^2}p+\left(b''_{y^2}+2A'_y\right)q+A''_{y^2}z\right]\right\}+$$
$$+2c\left\{p\left[\frac{\partial r}{\partial x}+\frac{\partial t}{\partial x}+ar+\left(a'_x+A\right)p+b'_x q+A'_x z\right]+\right.$$
$$\left.+q\left[\frac{\partial r}{\partial y}+\frac{\partial t}{\partial y}+bt+a'_y p+\left(b'_y+A\right)q+A'_y z\right]\right.$$

Donc,

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}-\frac{\partial u}{\partial \lambda}=2\lambda F\left[\left(\frac{\partial r}{\partial x}\right)^2+3\left(\frac{\partial r}{\partial y}\right)^2+3\left(\frac{\partial t}{\partial x}\right)^2+\left(\frac{\partial t}{\partial y}\right)^2\right]+$$
$$+2\lambda\left[\left(2F'_x-aF\right)\left(r\frac{\partial r}{\partial x}+t\frac{\partial t}{\partial x}\right)+\left(2F'_y-bF\right)\left(r\frac{\partial r}{\partial y}+t\frac{\partial t}{\partial y}\right)\right]+$$
$$+\lambda\left\{\left(F''_{x^2}+F''_{y^2}\right)\left(r^2+t^2\right)-2F\left[r^2\left(2a'_x+A\right)+t^2\left(2b'_y+A\right)\right]\right\}-$$
$$-2\lambda F\left\{r\left[\left(a''_{x^2}+2A'_x\right)p+b''_{x^2}q+A''_{x^2}z\right]+t\left[a''_{y^2}p+\left(b''_{y^2}+2A'_y\right)q+A''_{y^2}z\right]\right\}+$$
$$+2c\left[r^2+t^2-apr-bqt-\left(a'_x+A\right)p^2-\left(b'_x+a'_y\right)pq-\right.$$
$$\left.-\left(b'_y+A\right)q^2-z\left(A'_x p+A'_y q\right)\right]\leqslant 0.$$

Par conséquent, en tenant compte de l'inégalité:

$$F\left[\left(\frac{\partial r}{\partial x}\right)^2+\left(\frac{\partial r}{\partial y}\right)^2+\left(\frac{\partial t}{\partial x}\right)^2+\left(\frac{\partial t}{\partial y}\right)^2\right]+$$
$$+\left(2F'_x-aF\right)\left(r\frac{\partial r}{\partial x}+t\frac{\partial t}{\partial x}\right)+\left(2F'_y-bF\right)\left(r\frac{\partial r}{\partial y}+t\frac{\partial t}{\partial y}\right)\geqslant$$
$$\geqslant-\frac{1}{F}\left[\left(F'_x-\frac{1}{2}aF\right)^2+\left(F'_y-\frac{1}{2}bF\right)^2\right]\left(r^2+t^2\right),$$

et du fait que z, p, q sont bornés dans S_1, nous obtenons l'inégalité

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}-\frac{\partial u}{\partial \lambda}>\left(r^2+t^2\right)\left[2c-\frac{\lambda F}{2}\left(a+b+4A\right)+\right.$$
$$+2\lambda\left(aF'_x+bF'_y-\frac{F'^2_x+F'^2_y}{F}\right)+\lambda\left(F''_{x^2}+F''_{y^2}\right)\left.\right]-$$
$$-4\lambda F\left(a'_x r^2+b'_y t^2\right)+c\left(Pr+Qt+H\right),]$$

où P, Q, H sont bornés.

Donc, en prenant c assez grand pour que les coefficients de r^2 et t^2 deviennent supérieurs à c, nous voyons qu'au point, où u atteint son maximum, on a nécessairement

$$r^2+t^2+Pr+Qt+H\leqslant 0,$$

ou bien en posant $|P|\leqslant N_1$, $|Q|\leqslant N_1$, $|H|\leqslant N_2$,

$$r^2+t^2<2(N_1^2+N_2).$$

Ainsi, on a en tous les points de la région S_1:

$$r^2+2s^2+t^2\leqslant\frac{1}{\lambda_1(R_1^2-x^2-y^2)^2}\left[2TR_1^4(N_1^2+N_2)+\frac{M_1^2}{\lambda_0(R^2-R_1^2)^2}\right].$$

En observant que p, q entrent dans $N_1^2+N_2$ au degré non supérieur à deux, nous pouvons donc, en posant $\lambda_1=\lambda_0$, $R_1^2-x^2-y^2=R^2-R_1^2$, donner l'inégalité:

$$\sqrt{r^2+2s^2+t^2}<\frac{M_2}{\lambda[R^2-x^2-y^2]^2} \tag{5}$$

pour toute la région S, où M_2 est une même constante pour tous les points de cette région.

En appliquant la même méthode on pourrait au besoin limiter ainsi successivement d'une façon analogue les dérivées d'ordre supérieur.

Remarquons que par le fait les constantes M_1 et M_2 dans les inégalités (4), (5) dépendent du maximum de $|z|$ dans la région correspondante S. Mais dans tous les cas où la solution z de l'équation représente la limite d'une densité des probabilités, la méthode que j'ai signalée au début donne une limite supérieure de $|z|$ dans tout le plan de x, y pour toute valeur de $\lambda\,(0\leqslant\lambda\leqslant T)$ qui ne dépend que du maximum de $|z|$ pour $\lambda=0$.

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MATHÉMATIQUES

SUR QUELQUES GÉNÉRALISATIONS DU PROBLÈME DES DÉFORMATIONS INFINIMENT PETITES DES SURFACES DE ROTATION À COURBURE NÉGATIVE

Par V. S. LUKCHIN

(Présenté par I. M. Vinogradow, de l'Académie, le 9. I. 1938)

1. Dans un article sur la déformation des surfaces à courbure négative ayant un point singulier (1) nous avons considéré une surface dont la ligne frontière est parabolique. D'une manière analogue à celle de l'étude des surfaces régulières (2), nous allons étudier maintenant la déformation d'une surface de rotation à courbure négative possédant d'un côté un point singulier conique et de l'autre côté une frontière ordinaire.

Supposons que dans le plan (z, r) l'arc OA est défini par l'équation:

$$r = r(z) = az + \varphi(z), \tag{1}$$

où

$$a > 0, \quad \varphi(0) = \varphi'(0) = 0, \quad \varphi''(0) > 0, \quad \varphi'(z) > 0, \quad \varphi''(z) > 0,$$

$$\varphi(z_0) > 0, \ \varphi'(z_0) = \theta > 0; \quad 0 \leqslant z \leqslant z_0, \ r_0 = az_0 + \varphi(z_0); \quad r_0' = a + \theta.$$

Désignons par S la surface de rotation qu'on obtient en faisant tourner cette ligne autour de l'axe Oz. Cherchons la surface S_1 que l'on obtient de S par déformation, t étant le paramètre de déformation:

$$\left.\begin{aligned} z_1 &= z + tZ(z, v), \\ r_1 &= r(z) + tR(z, v), \\ v_1 &= v + tV(z, v). \end{aligned}\right\} \tag{2}$$

Les fonctions cherchées Z, R, V sont développables en série de Fourier [voir mon article (1)] et s'obtiennent des équations suivantes

$$\begin{aligned} r'R_n' + Z_n' &= 0, \\ (n^2 - 1)r'R_n + rR_n' + n^2 Z_n &= 0, \end{aligned} \tag{3}$$

d'où en excluant Z_n on trouve:

$$rR_n'' + (n^2 - 1)r''R_n = 0. \tag{4}$$

Le seul point singulier de cette équation $z = 0$ sera un point ordinaire pour les solutions de la même équation (4).

Le problème de déformation ne peut être résolu que par l'intégrale particulière de la forme:

$$R_n(z) = b_0 z g(z) \qquad (5)$$

où $b_0 \neq 0$, $g(0) = 1$, $g(z)$ est une fonction holomorphe.

Le point singulier $O(0, 0)$ reste fixe pendant la déformation et ne cesse pas d'être singulier, à savoir conique. La frontière ordinaire de la surface S, un cercle de rayon r_0, se transforme pendant la déformation en une ligne gauche, ligne frontière de la surface S_1. Nous cherchons la déformation de S dans les conditions suivantes.

a) Trouver une surface S_1, dont la ligne frontière est située sur le cylindre ayant pour base le cercle frontière de la surface S et pour génératrices des droites parallèles à l'axe Oz. La solution du problème doit vérifier la condition

$$R_n(z_0) = 0.$$

La solution (5) pour un $n = n_0 > 1$ donné vérifiera cette condition pour certaines valeurs du paramètre a:

$$a = a^{(1)} < a^{(2)} < \ldots < a^{(i)} < a^{(i+1)} < \ldots < a^{(m)}.$$

b) Trouver une surface S_1 dont la ligne frontière est une courbe plane. Parmi les surfaces S de la famille on pourra trouver des surfaces dont les lignes frontières deviennent, après la déformation, des courbes planes situées dans le plan $z = z_0$. Les surfaces cherchées S pourront être trouvées de la condition

$$Z_n(z_0) = 0, \qquad (6)$$

quel que soit n. Il suit de la seconde équation du système (3) que l'on a

$$\frac{R_n'(z_0)}{R_n(z_0)} = -\frac{(n^2-1)r_0'}{r_0},$$

où le second membre peut être présenté dans la forme:

$$-\frac{(n^2-1)}{z_0}\left[1 + \frac{z_0\theta - \varphi(z_0)}{az_0 + \varphi(z_0)}\right].$$

Quand a augmente en variant entre $a^{(i)}$ et $a^{(i+1)}$, ce rapport, tout en restant borné, varie en un sens, et comme le rapport dans le premier membre varie entre $-\infty$ et $+\infty$, la condition (6) sera vérifiée pour $n = n_0$ pour une valeur $a = \bar{a}^{(i)}$ située dans l'intervalle

$$a^{(i)} < \bar{a}^{(i)} < a^{(i+1)}.$$

c) Parmi les surfaces, sur lesquelles peut être située la ligne frontière de la surface S, il existe des cônes dont les directrices coïncident avec le cercle frontière de la surface S, tandis que leurs sommets se trouvent sur l'axe Oz. En coordonnées cylindriques l'équation d'un tel cône peut être écrite dans la forme:

$$r_1 - r_0 = \gamma(z_1 - z_0),$$

où z_1, r_1 sont les coordonnées courantes, γ—le coefficient angulaire de la génératrice rectiligne du cône.

Nous cherchons une surface S dont la ligne frontière est située sur ce cône; à cet effet il faut que la condition suivante soit remplie:

$$R_n(z_0) = \gamma Z_n(z_0),$$

ou bien, en vertu de (3), la condition:

$$\frac{R_n'(z_0)}{R_n(z_0)} = -\frac{(n^2-1)(\theta+a)\gamma+n^2}{r_0\gamma}.$$

Pour un certain $n=n_0$ nous trouverons des valeurs $a=\bar{a}^{(i)}$ situées dans l'intervalle $[a^{(i)}, \bar{a}^{(i)}]$ ou bien $[\bar{a}^{(i)}, a^{(i+1)}]$ selon le signe de γ, et d'ailleurs pour ces $\bar{a}^{(i)}$ la ligne frontière de S se trouvera sur le cône.

Il existe pour chaque cône des nombres $\bar{\bar{a}}^{(i)}$, en faisant varier n et a nous pouvons effectuer la déformation de S quel que soit le cône donné. En particulier si $\gamma=0$ le cône devient un cylindre, alors $\bar{\bar{a}}^{(i)}=a^{(i)}$ ou bien $\bar{\bar{a}}^{(i)}=a^{(i+1)}$; pour $\gamma=\pm\infty$ le cône devient le plan du cercle frontière de la surface S, alors on a $\bar{\bar{a}}^{(i)}=\bar{a}^{(i)}$.

2. Dans nos recherches précédentes ainsi que dans les travaux de Rembs [3] on suppose que la courbe formant la surface de rotation est de courbure partout positive. On peut rejeter cette restriction. Il y a plus: on peut supposer que sur un arc assez petit la courbe a une courbure négative, pourvu que le rebord qu'on obtient alors sur la courbe soit assez petit en profondeur et largeur et que la tangente à la courbe en variant d'une manière continue ne devienne jamais perpendiculaire à l'axe de rotation. Dans ces hypothèses tous les théorèmes sur la déformation des surfaces de rotation subsistent, c'est-à-dire la surface S ayant un pareil épaississement le long d'un certain cercle de latitude est déformable ou non déformable, ce qui dépend du paramètre a correspondant. En faisant varier d'une manière analogue une surface convexe de rotation, qui n'admet aucune déformation, on obtient une surface de rotation ayant un enfoncement assez petit le long d'un cercle de latitude, et une telle surface admet une déformation infiniment petite du premier ordre ainsi qu'il a été démontré par Cohn-Vossen [4].

L'équation (4) peut être représentée dans la forme suivante:

$$R_n'' + G(z)R_n = 0, \qquad G(z) = \frac{(n^2-1)r''}{r}. \qquad (7)$$

Supposons que l'arc OA a deux points d'inflexion $B(\bar{z}_1)$ et $C(\bar{z}_2)$ et que l'arc suffisamment petit BC tourne sa concavité vers l'axe Oz; pour le rebord obtenu on a: $G(z)<0$.

Pour démontrer le théorème précédemment mentionné il faut établir les théorèmes de Sturm-Liouville dont nous profitons en étudiant les zéros des solutions de l'équation (7). Considérons deux équations:

$$y''+G_1y=0 \quad \text{et} \quad y''+G_2y=0,$$

dont les intégrales sont respectivement $y_1(z)$ et $y_2(z)$. Supposons que $G_2(z)>G_1(z)>0$ partout, sauf pour l'intervalle $[\alpha, \beta]$, où $G_2\leqslant G_1$; d'ailleurs $\alpha<\bar{z}_1<\bar{z}_2<\beta$; soit $G_2(z)<0$ dans l'intervalle $(\bar{z}_1, \bar{z}_2)$. Supposons que $y_1(z)$ a deux zéros consécutifs z_1 et z_2 et que l'intervalle $[\alpha, \beta]$ se trouve à l'intérieur de $[z_1, z_2]$. On peut alors démontrer le théorème d'oscillation, c'est-à-dire démontrer qu'entre deux zéros consécutifs de y_1 il se trouve nécessairement ou moins un zéro de y_2; à cet effet on présente la formule de Sturm:

$$\left[y_1'y_2 - y_1y_2'\right]_{z_1}^{z_2} = \int_{z_1}^{z_2}(G_2-G_1)y_1y_2dz,$$

dans la forme:

$$\int_{z_1}^{\alpha} + \int_{\alpha}^{\beta} + \int_{\beta}^{z_2},$$

et on prend l'intervalle $[\alpha, \beta]$ assez petit pour que le second membre soit positif; le premier membre est inférieur ou égal à zéro [Bôcher (5)].

En comparant l'équation (7) avec l'équation $Q'' + \lambda^2 Q = 0$, $\lambda = \text{const}$, $Q = \sin \lambda z$ nous voyons que la solution $R_n(z)$ de l'équation (7) possède dans $[0, z_0]$ autant de zéros que l'on veut. En faisant a croître, on déplace les zéros de $R_n(z)$, et pour $a = a^{(i)}$ la déformation de S aura lieu, la ligne frontière, d'ailleurs, devient une courbe sur le cylindre correspondant. Comme le second théorème de comparaison est aussi vrai, il existe parmi les surfaces ayant un épaississement le long d'un cercle de latitude des surfaces qui admettent la déformation dans les conditions b) c) du § 1.

3. Il est facille de rejeter l'hypothèse que la ligne OA est analytique pour les parties de l'intervalle $[0, z_0]$ dont les points ne sont pas des points singuliers pour l'équation différentielle. Supposons que l'arc OA possède partout une courbure continue et que sur un certain intervalle au voisinage de l'origine la partie correspondante de la courbe est analytique. On peut alors appliquer à cette partie de la courbe la théorie de Fuchs, ce qui permet d'étudier l'allure de l'intégrale de l'équation (4) au voisinage du point singulier de cette équation différentielle. Comme pour tout cet intervalle la théorie de Sturm-Liouville est applicable [Bôcher (5)], les théorèmes sur la déformabilité ou la non-déformabilité de la surface de rotation formée par l'arc OA restent valables.

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MATHÉMATIQUES

SUR LES FONCTIONS SATISFAISANT AUX ÉQUATIONS DU TYPE ELLIPTIQUE

Par B. GAGAEFF

(Présenté par S. N. Bernstein, de l'Académie, le 7. I. 1938)

Considérons les familles des fonctions satisfaisant aux équations du type elliptique:

$$A\frac{\partial^2 u}{\partial x^2}+2B\frac{\partial^2 u}{\partial x\partial y}+C\frac{\partial^2 u}{\partial y^2}+D\frac{\partial u}{\partial x}+E\frac{\partial u}{\partial y}+F(u)=0 \qquad (1)$$
$$AC-B^2>0,$$

où A, B, C possèdent des dérivées partielles jusqu'au troisième ordre, D, E—jusqu'au second ordre et F—du premier ordre, et cherchons des critères pour que ces familles soient normales. L. Lichtenstein [1] a démontré que dans ce cas on peut étendre le théorème de Harnack aux fonctions satisfaisant à l'équation (1). Il a démontré que si $u_m(x, y)$ sont des intégrales régulières (c'est-à-dire continues ainsi que leur dérivées partielles jusqu'au second ordre) de l'équation (1), positives partout dans un domaine connexe T limité par une courbe S à courbure continue, et si la série

$$\sum_{k=1}^{\infty} u_k(x, y) \qquad (2)$$

converge en un point intérieur à T, alors elle converge uniformément partout à l'intérieur d'un domaine connexe T' complètement intérieur à T et limité par une courbe à courbure continue; sa somme est une intégrale régulière de l'équation (1).

Lichtenstein a déduit ce théorème de lemme suivant: *Si* $u_m(x, y)$ est une intégrale régulière de l'équation (1), satisfaisant aux mêmes conditions, et x_1, y_1 et x_2, y_2 sont deux points intérieurs à T', on a:

$$\frac{1}{\lambda}u_m(x_1, y_1) > u_m(x_2, y_2) > \lambda u_m(x_1, y_1),$$

où λ est un nombre positif intérieur à l'unité, qui dépend seulement de T, T' et des coefficients de l'équation (1) et ne dépend pas de l'intégrale $u_m(x, y)$.

On déduit de ce lemme le théorème suivant:

Théorème 1. *Si* $u_m(x, y)$ *sont des intégrales régulières de l'équation du type elliptique et positives (négatives) partout dans un domaine* T, *elles forment une famille normale dans ce domaine.*

En effet, dans ce cas toute suite infinie des valeurs des fonctions en un point x_1, y_1:

$$u_{m_1}(x_1, y_1), \quad u_{m_2}(x_1, y_1), \quad \ldots, \quad u_{m_k}(x_1, y_1), \quad \ldots$$

converge ou diverge. Dans le premier cas la suite

$$u_{m_1}(x, y), \quad u_{m_2}(x, y), \quad \ldots, \quad u_{m_k}(x, y)$$

converge uniformément partout dans chaque domaine T', dans le second cas elle converge uniformément vers l'infini. Le théorème est démontré.

On peut déduire de ce théorème les critères suivants qui permettent de reconnaître, si une famille est normale.

Théorème 2. *Si une famille d'intégrales régulières d'une équation du type elliptique est uniformément bornée inférieurement (supérieurement) partout dans un domaine T, alors elle est normale dans ce domaine.*

En effet, soient $u_m(x, y)$ les intégrales uniformément bornées inférieurement dans T, c'est-à-dire partout dans T $u_m(x, y) > A$. Si A est positif ou nul, la famille est normale. Soit $A < 0$ et soit T' un domaine complètement intérieur à T et soit x_2, y_2 un point intérieur à T, on a:

$$u_m(x_2, y_2)) = \frac{1}{2\pi} \int_{S'} \frac{\partial H(x_2, y_2; x_1, y_1)}{\partial n} u_m(x_1, y_1)\, ds,$$

où S est le contour de T' et $H(x_2, y_2; x_1, y_1)$ est la fonction de Green de l'équation adjointe, qui s'annule sur le contour S'.

Lichtenstein [2] a démontré que sur le contour S' nous avons

$$\frac{\partial H(x_2, y_2; x_1, y_1)}{\partial n} > \alpha > 0$$

où α ne dépend ni du point x_1, y_1, ni du point x_2, y_2. Par conséquent, la fonction

$$u(x_2, y_2) = -\frac{1}{2\pi l \alpha} \int_{S'} \frac{\partial H(x_2, y_2; x_1, y_1)}{\partial n} A\, ds,$$

où l est la longueur de S', est l'intégrale régulière de l'équation (1), qui satisfait partout dans T' à la condition

$$u(x_2, y_2) > -A.$$

Par conséquent, la famille $u_m(x, y) + u(x, y)$ est positive partout dans T'. Ainsi, d'après le théorème 1, elle est normale dans T' et d'après le théorème des familles normales il en est de même dans T.

Théorème 3. *Si une famille d'intégrales régulières de l'équation du type elliptique ne prend jamais une valeur α dans un domaine T, alors cette famille est normale dans T.*

En effet, on peut extraire de toute suite infinie de fonctions de la famille

$$u_{m_1}(x, y), \quad u_{m_2}(x, y), \quad \ldots$$

une suite infinie de fonctions qui sont supérieures à α partout dans T' ou bien une suite infinie de fonctions qui sont inférieures à α partout dans T. Dans le premier cas nous avons une suite infinie de fonctions bornées inférieurement, dans le second cas nous avons une suite infinie de fonctions bornées supérieurement. Ainsi la famille est normale.

On ne peut pas étendre les théorèmes démontrés aux cas des fonctions qui satisfont à une équation du type hyperbolique. En effet, la famille

$$\varphi_m(x, y) = m(x^2 + y^2) + 1 \qquad (m > 0).$$

est positive partout et satisfait à l'équation du type hyperbolique

$$\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} = 0.$$

Cependant dans tout le domaine, qui contient le point 0,0, elle n'est pas normale, car au point 0,0 toute la suite converge vers 1 et en tous les autres points elle croît indéfiniment. Le même exemple montre qu'on ne peut pas étendre les théorèmes 2 et 3 aux intégrales d'une équation du type hyperbolique.

Les fonctions harmoniques et polyharmoniques qui satisfont partout à un certain critère de normalité sont constantes. On ne peut pas étendre cette propriété aux intégrales d'une équation du type elliptique. Par exemple les fonctions $A \cos x$ satisfont à l'équation:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + u = 0,$$

et sont bornées partout.

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MATHÉMATIQUES

LA REPRÉSENTATION ANALYTIQUE DE QUELQUES FONCTIONNELLES PARTIELLEMENT ADDITIVES

Par A. G. PINSKER

(Présenté par S. N. Bernstein, de l'Académie, le 19. I. 1938)

Nous désignerons par fonctionnelle «partiellement additive» la fonctionnelle $I(x)$, si la relation d'additivité $I(x+y)=I(x)+I(y)$ n'a lieu que pour certaines paires d'éléments x, y satisfaisant à une condition déterminée. Les fonctionnelles partiellement additives et continues présentent une généralisation immédiate des fonctionnelles linéaires.

Dans cette note nous indiquerons les formes générales de quelques fonctionnelles continues et partiellement additives (en plusieurs sens) dans l'espace L des fonctions $x(t)$ intégrables au sens de Lebesgue, définies dans un segment $a \leqslant t \leqslant b$ et dans l'espace l des suites de nombres $x=\{\xi_i\}$ tels que

$$\sum_{i=1}^{\infty} |\xi_i| < +\infty.$$

La solution des problèmes de ce genre dans beaucoup de cas est devenue possible grâce à la théorie des espaces réguliers semi-ordonnés construite par M. L. Kantorovitch (1). Les résultats et les méthodes de cette théorie (appliqués à l'espace L) sont utilisés dans la démonstration des théorèmes qui suivent.

Avant d'examiner les fonctionnelles dans l'espace L il convient de démontrer le théorème suivant, qui est lui-même important.

Théorème 1. a) Si la fonctionnelle

$$I(x)=\int_a^b f[x(t), t]\,dt \qquad (*)$$

est continue dans L, la fonction $f(p, t)$ $(-\infty < p < +\infty,\ a \leqslant t \leqslant b$ doit satisfaire aux conditions suivantes:

1. Pour toute valeur fixe de p, $f(p, t) \in L$.
2. Pour toute valeur fixe de p, $|f(p, t)| \leqslant K|p| + \Phi(t)$ presque partout $(K=\text{const},\ \Phi(t) \in L)$.
3. Quel que soit $\varepsilon > 0$, il existe δ tel que l'inégalité:

$$|p' - p''| < \delta \qquad (p_1 \leqslant p',\ p'' \leqslant p_2),$$

implique $|f(p', t) - f(p'', t)| < \varepsilon \cdot R(t)$ presque partout, où $R(t)$ est une fonction intégrable, qui ne dépend que du choix des nombres p_1 et p_2 et qu'on peut nommer «régulateur de la continuité uniforme».

b) Si la fonction $F(p, t)$ est continue par rapport à p et satisfait aux conditions 1 et 2, la fonctionnelle

$$I(x) = \int_a^b F[x(t), t]\, dt \qquad (**)$$

est continue dans L.

c) Toute fonctionnelle continue dans L, pouvant être représentée sous la forme (*), peut être représentée de même sous la forme (**), la fonction $f(p, t)$ coïncidant, presque partout pour tout p fixe, avec la fonction $F(p, t)$.

Indiquons les moments essentiels de la démonstration du théorème.

a) Il est évident que la condition 1 est nécessaire. La condition 2 équivaut à l'existence d'une constante K, vérifiant la condition:

$$\sup_p \{\max[|f(p, t)| K|p|] - K|p|\} \in L.$$

En supposant le contraire, on verrait que pour toutes les valeurs de i $(i = 1, 2, \ldots)$ les ensembles de fonctions A_i dépendant du paramètre p:

$$A_i = \{\max[|f(p, t)|, K_i|p|] - K_i|p|\} \qquad (K_i = 2^i),$$

sont non bornés. (La terminologie «ensemble borné», «supremum», «infimum» est celle de la théorie des espaces linéaires semi-ordonnés.)

Vu la régularité de l'espace L, on peut extraire des ensembles A_i un nombre fini de fonctions:

$$\varphi_i^{(1)}(t), \varphi_i^{(2)}(t), \ldots \varphi_i^{(n_i)}(t) \qquad (i = 1, 2, \ldots),$$

remplissant les conditions suivantes:

a) $|f(p_i^{(k)}, t)| = K_i|p_i^{(k)}| + \varphi_i^{(k)}(t)$ $(t \in e_i^{(k)})$

les ensembles $e_i^{(k)}$ n'ayant pas de points communs, $\sum e_i^{(k)} = E$, mes $E > 0$.

b) $\varphi_i^{(k)}(t) \geqslant f(t)$ $(t \in e_i^{(k)})$, où $f(t)$

est une fonction non intégrable dans l'ensemble E.

Deux cas peuvent se présenter:

1) La série $\sum_{i,k} |p_i^{(k)}| \operatorname{mes} e_i^{(k)} < +\infty$ et

2) $\sum_{i,k} |p_i^{(k)}| \operatorname{mes} e_i^{(k)} = +\infty$.

Dans le premier cas posons $x(t) = p_i^{(k)}$ dans l'ensemble $e_i^{(k)}$ et $x(t) = 0$ partout ailleurs, $x(t) \in L$, mais

$$\int_E |f[x(t), t]|\, dt = \sum_{i,k} \int_{e_i^{(k)}} |f(p_i^{(k)}, t)|\, dt \geqslant \int_E f(t)\, dt = +\infty$$

ce qui est impossible.

Dans le second cas on peut indiquer des ensembles $\mathcal{E}_{i_s}^{(k)} \subset e_{i_s}^{(k)}$ tels que

$$\sum_{k=1}^{n_{i_s}} |p_{i_s}^{(k)}| \operatorname{mes} \mathcal{E}_{i_s}^{(k)} = \frac{1}{2^{i_s}} \qquad (s = 1, 2, \ldots).$$

Soit $x(t)=p_{i_s}^{(k)}$ dans l'ensemble $\mathcal{E}_{i_s}^{(k)}$ et soit $x(t)=0$ partout ailleurs, $x(t)\in L$; mais

$$\int_{\mathcal{E}=\sum\mathcal{E}_{i_s}^{(k)}} |f[x(t),t]\,dt| =$$

$$=\sum_{s=1}^{\infty}\Big(\sum_{k=1}^{n_{i_s}}\int_{\mathcal{E}_{i_s}^{(k)}}|f(p_{i_s}^{(k)},t)|\,dt\Big)>\sum_{s=1}^{\infty}\Big(\sum_{k=1}^{n_{i_s}}K_{i_s}|p_{i_s}^{(k)}|\operatorname{mes}\mathcal{E}_{i_s}^{(k)}\Big)=$$

$$=\sum_{s=1}^{\infty}\Big(2^{i_s}\sum_{k=1}^{n_{i_s}}|p_{i_s}^{(k)}|\operatorname{mes}\mathcal{E}_{i_s}^{(k)}\Big)=\sum_{s=1}^{\infty}2^{i_s}\,\frac{1}{2^{i_s}}=+\infty$$

ce qui est absurde.

Pour démontrer la condition 3 décomposons l'intervalle (p_1, p_1) en 2^n parties égales; l'ensemble $\{f(p,t)\}$ où p appartient à l'un des intervalles partiels, a un supremum et un infimum. Désignons leur différence par $\varphi_k(t)$ $(k=1, 2, \ldots 2^n)$: soit $\psi_n(t)=\max_k\{\varphi_k(t)\}$ $(n=1, 2, \ldots)$.

La suite des fonctions non-négatives $\psi_n(t)$ tend monotonnement vers zéro en décroissant (ce qu'on peut prouver en s'appuyant sur les propriétés des espaces réguliers et en appliquant le théorème de Egoroff), il existe donc (d'après le théorème sur la convergence dans les espaces réguliers) une fonction intégrable $R(t)$ telle que $|\psi_n(t)|\leqslant\varepsilon R(t)$ presque partout pour $n\geqslant N_\varepsilon$ (ε étant aussi petit que l'on veut) d'où l'on obtient facilement 3.

b) Peut être démontré à l'aide du lemme:

Si $F(p,t)$ est continue par rapport à p et satisfait aux conditions 1, 2 et 3, on a: $|F[x(t),t]\leqslant K|x(t)|+\Phi(t)$ presque partout ($K=\text{const}$, $\Phi(t)\in L$). On déduit c) de b) à l'aide du lemme qui suit:

Si $f(p,t)$ satisfait aux conditions 1, 2 et 3, il existe une fonction $F(p,t)$ continue par rapport à p coïncidant avec $f(p,t)$ presque pour tout p fixe.

Remarque. Si la fonctionnelle (*) est continue dans L, on aura:

$$I(x)=\lim_{\alpha\to 0}\sum_{i=-\infty}^{+\infty}\int_{e_i}f(p_i,t)\,dt,$$

où $\ldots<p_{-1}<p_0<p_1<\ldots$ est une suite de nombres infinie dans les deux sens, $p_{i+1}-p_i<\alpha$, $e_i=\mathcal{E}_t[p_i\leqslant x(t)<p_{i+1}]$ sont des ensembles de Lebesgues des fonctions $x(t)$.

Théorème 2. La forme générale d'une fonctionnelle continue dans L, partiellement additive dans ce sens que

$$I(x+y)=I(x)+I(y),\ \text{si}\ x(t)\cdot y(t)=0\quad (x=x(t),\ y=y(t))$$

est donnée par la formule:

$$I(x)=\int_a^b F[x(t),t]\,dt$$

où $F(p,t)$ est continue par rapport à p et satisfait aux conditions 1 et 2 du théorème 1, a) et à la condition complémentaire:

3) $F(0,t)=0$ presque partout.

Soit $x(t)=p$ dans un ensemble (mesurable) e et soit $x(t)=0$ partout ailleurs; posons $I(x)=\Phi_p(e)$; il est aisé de voir que $\Phi_p(e)$ est une fonction additive et absolument continue comme une fonction de l'ensemble e Il existe une fonction intégrable $f(p, t)$ telle que

$$\int_e f(p, t)\,dt=\Phi_p(e) \qquad (-\infty<p<+\infty).$$

En répétant le raisonnement utilisé dans la démonstration du théorème 1, on conclut que la fonction $f(p, t)$ satisfait nécessairement aux conditions 1, 2 et 3 du théorème 1, a) et qu'il existe une fonction $F(p, t)$ continue par rapport à p, coïncidant avec la fonction $f(p, t)$ presque partout quel que soit p.

D'autre part, la fonctionnelle continue dans L.

$$I_1(x)=\int_a^b F[x(t), t]\,dt$$

est égal à $I(x)$, si $x(t)$ est une fonction à un nombre fini de valeurs distinctes.

En effet, soit $x(t)=p_i$ $(t\in e_i,\ i=1, 2, \ldots n)$; alors en vertu de l'additivité de la fonctionnelle on a:

$$I(x)=\sum_{i=1}^{n}\Phi_{p_i}(e_i)=\sum_{i=1}^{n}\int_{e_i} f(p_i, t)\,dt,$$

mais

$$I_1(x)=\sum_{i=1}^{n}\int_{e_i} F(p_i, t)\,dt=\sum_{i=1}^{n}\int_{e_i} f(p_i, t)dt$$

c'est-à-dire, $I(x)=I_1(x)$. Les fonctionnelles continues dans L coïncidant pour les fonctions à un nombre fini de valeurs distinctes sont identiquement égales. La condition 3 est une conséquence de l'additivité de la fonctionnelle.

Théorème 3. La forme générale d'une fonctionnelle continue dans L, partiellement additive dans ce sens que

$$I(x+y)=I(x)+I(y), \text{ si } x(t)\cdot y(t)\geqslant 0 \quad (x=x(t),\ y=y(t))$$

est donnée par la formule:

$$I(x)=\Phi_1[(x)_+]+\Phi_2[(x)_-],$$

où Φ_1 et Φ_2 sont des fonctionnelles linéaires, $(x)_+=[x(t)]_+=\max[0, x(t)]$, $(x)_-=[x(t)]_-=\max[0, -x(t)]$.

Théorème 4. La fonctionnelle $I(x)$ continue dans L partiellement additive dans ce sens que

$$I(x+y)=I(x)+I(y), \text{ si } x(t)\cdot y(t)\leqslant 0 \quad (x=x(t),\ y=y(t))$$

est une fonctionnelle linéaire.

Théorème 5. La forme générale d'une fonctionnelle continue dans L ayant les propriétés suivantes:

a) $I(x+y)=I(x)+I(y)$, si $x(t)\cdot y(t)=0$ $(x=x(t),\ y=y(t))$;

b) $I(x)=I(y)$, si $x(t+h)=y(t)$ (en supposant qu'en dehors de l'intervalle (a, b) les fonctions $x(t)$ et $y(t)$ s'annulent), est donnée par la formule

$$I(x)=\int_a^b F[x(t)]\,dt,$$

où la fonction $F(p)$:

1) est continue;

2) $|F(p)|<K|p|+C$ (K et C étant des constantes);

3) $F(0)=0$.

Théorème 6. La forme générale d'une fonctionnelle continue dans l et partiellement additive, ce qui signifie maintenant que $I(x+y)=I(x)+I(y)$, si les éléments $x=\{\xi_i\}$ et $y=\{\eta_i\}$ satisfont à la relation $\xi_i\cdot\eta_i=0$ $(i=1, 2, \ldots)$, est donnée par la formule:

$$I(x)=\sum_{i=1}^{\infty}\varphi_i(\xi_i),$$

où $x=\{\xi_i\}$ et les fonctions $\varphi_i(\xi)$ satisfont aux conditions:

1) $\varphi_i(\xi)$ sont continues et $\varphi_i(0)=0$ $(i=1, 2, \ldots)$;

2) $|\varphi_i(\xi)|<K|\xi|+C_i$ où $|\xi|<\delta$ (K et δ étant des constantes, $\sum_{i=1}^{\infty}|C_i|<+\infty$).

Finalement, on peut démontrer pour l'espace l les théorèmes analogues aux théorèmes 3 et 4 énoncés pour L.

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MATHÉMATIQUES

SUR LES ENSEMBLES RÉGULIÈREMENT FERMÉS ET FAIBLEMENT COMPACTS DANS L'ESPACE DU TYPE (B)

Par V. ŠMULIAN

(Présenté par S. N. Bernstein, de l'Académie, le 17. I. 1938)

La note considérée présente le développement des résultats obtenus par V. Gantmakher et l'auteur dans leur article commun (1). D'ailleurs, on trouve ici la réponse aux quelques questions posées par les auteurs de l'article cité.

1. Soit E un espace du type (B) (2). Nous disons que la suite $\{x_n\} \subset E$ converge vers x_0 en moyenne, si on a:

$$\underline{\lim} f(x_n) \leqslant f(x_0) \leqslant \overline{\lim} f(x_n) \text{ pour chaque } f \in \bar{E}.$$

Notons d'abord la proposition suivante:

Théorème 1. *Pour qu'un ensemble $\mathfrak{M} \subset E$ soit faiblement compact il faut et il suffit que chaque suite $\{x_n\} \subset \mathfrak{M}$ converge en moyenne vers un élément $x_0 \in E$.*

Démonstration. Il est évident que la condition est nécessaire; il s'agit de démontrer qu'elle est suffisante. Sans restreindre la généralité, nous pouvons supposer que l'espace E est séparable. Alors il existe une suite $\{f_m\}$ des fonctionnelles linéaires faiblement dense dans $\bar{E}$.

Considérons une suite quelconque $\{x_n\} \subset \mathfrak{M}$. Il est évident que cette suite contient toujours une suite partielle $\{x_{n_\nu}\}$ telle que la limite $\lim f(x_{n_\nu})$ existe pour tout $f = f_m$. Posons $y_\nu = x_{n_\nu}$ $(\nu = 1, 2, \ldots)$.

Par hypothèse, la suite $\{y_\nu\}$ converge en moyenne vers un élément x_0. Nous allons montrer que la suite $\{y_\nu\}$ converge faiblement vers x_0. En admettant le contraire on peut affirmer, qu'il existe une fonctionnelle $f_0 \in \bar{E}$ telle que la suite $\{f_0(y_\nu)\}$ contient une suite partielle $\{f_0(y_{\nu_k})\}$ convergeant vers une limite (finie ou infinie) différente de $f_0(x_0)$.

Posons $z_k = y_{\nu_k}$ $(k = 1, 2, \ldots)$. On obtient alors:

$$\lim_k f_m(z_k) = f_m(x_0) \tag{1}$$

pour $m = 1, 2, \ldots$ et $\lim_k f_0(z_k) \neq f_0(x_0)$.

Par hypothèse, la suite $\{z_k\}$ converge aussi en moyenne vers un élément x_0'.

En vertu de (1) nous avons:

$$f_m(x_0) = \lim_k f_m(z_k) = f_m(x_0') \tag{2}$$

pour $m = 1, 2, \ldots$, et

$$f_0(x_0') = \lim_k f_0(z_k) \neq f_0(x_0). \tag{3}$$

La suite $\{f_m\}$ étant faiblement dense dans $\bar{E}$, il suit de (2) que $f(x_0') = f(x_0)$ pour toute $f \in \bar{E}$, ce qui est impossible d'après (3).

Le cas particulier de ce théorème où $\mathfrak{M}$ est la sphère unitaire de E, a été établi auparavant [1] par une méthode indirecte.

Théorème 2. *Pour que l'espace séparable $F \subset \bar{E}$ soit régulièrement fermé il faut et il suffit que sa sphère unitaire soit faiblement compacte dans cet espace (au sens de la convergence faible des fonctionnelles).*

Démonstration. Soit F un sous-espace de $\bar{E}$ régulièrement fermé et séparable. Désignons par G l'ensemble des éléments $x \in E$ pour lesquels $f(x) = 0$, quel que soit $f \in F$.

En vertu d'une remarque de M. Krein et de l'auteur, F est l'espace conjugué à l'espace $\frac{E}{G}$. F étant séparable $\frac{E}{G}$ l'est également et la sphère unitaire dans F est alors faiblement compacte au sens de la convergence faible des fonctionnelles par rapport à l'espace $\frac{E}{G}$. Il ne reste qu'à remarquer que la convergence faible des fonctionnelles dans F par rapport à l'espace $\frac{E}{G}$ est équivalente à la convergence faible par rapport à l'espace E.

Pour démontrer que la condition du théorème est suffisante considérons d'abord la proposition générale suivante:

Quel que soit le sous-espace séparable $H \subset \bar{E}$, on peut trouver un sous-espace séparable $G \subset E$ tel que $H \subset G$.

Nous omettons la démonstration (d'ailleurs très simple) de cette proposition.

Soit à présent $f_0 \subset \bar{E}$ une fonctionnelle arbitraire en dehors de F. Prenons pour $H \subset \bar{E}$ un sous-espace linéaire fermé séparable contenant F et f_0. On a alors $F \subset H \subset \bar{G}$.

Si F satisfait à la condition du théorème il est faiblement fermé (au sens de la convergence faible des fonctionnelles dans G).

Comme G est d'ailleurs séparable, on en conclut d'après un théorème de M. Banach [3] que F est régulièrement fermé dans $\bar{G}$; il existe alors un élément $x_0 \in G$ tel que $f(x_0) = 0$ pour tout $f \in F$ et $f_0(x_0) = 1$.

Remarque. Il est facile de voir qu'on peut remplacer la condition du théorème 2 par la condition suivante: à toute suite bornée $\{f_m\} \subset F$ correspond une fonctionnelle $f_0 \in F$ telle que

$$\underline{\lim} f_m(x) \leqslant f_0(x) \leqslant \overline{\lim} f_m(x) \text{ pour tout } x \in E.$$

Nous pouvons maintenant démontrer la proposition suivante:

Théorème 3. *La propriété de la sphère $|x| \leqslant 1$ d'être faiblement compacte est équivalente à chacune des conditions suivantes.*

1. *Pour qu'un système arbitraire d'équations $f_\nu(x) = C_\nu (\nu = 1, 2, \ldots)$ ait une solution $x = x_0$ telle que $|x_0| \leqslant \mu$, il faut et il suffit que l'inégalité:*

$$\left| \sum_1^n C_\nu \cdot \lambda_\nu \right| \leqslant \mu \cdot \left| \sum_1^n f_\nu \cdot \lambda_\nu \right|,$$

ait lieu pour toute suite finie $\lambda_1, \ldots, \lambda_n$ de nombres réels.

II. *Chaque sous-espace linéaire* $\Gamma \subset \overline{E}$ *fermé et séparable est régulièrement fermé.*

Démonstration. La nécessité de ces conditions est connue [1]; pour démontrer qu'elles sont aussi suffisantes remarquons d'abord que la condition I implique la condition II. En effet, soit F un sous-espace fermé et séparable de $\overline{E}$ et $\{f_n\}$ — une suite dense par rapport à F. Soit f_0 une fonctionnelle quelconque en dehors de F. Considérons le système des équations:

$$f_0(x)=1,\ f_1(x)=0,\ f_2(x)=0,\ldots$$

Ce système a une solution x_0, si la condition I est remplie, et par suite F est régulièrement fermé.

Il ne reste qu'à démontrer que la condition II implique la propriété de la sphère $|x|\leqslant 1$ d'être faiblement compacte.

Soit $\{f_n\} \subset \overline{E}$ une suite bornée quelconque de fonctionnelles. Formons un sous-espace F linéaire fermé et séparable, contenant $\{f_n\}$. D'après la condition II F est régulièrement fermé et par conséquent $F=\left(\frac{\overline{E}}{\overline{G}}\right)$. D'autre part, tout sous-espace fermé $F_1 \subset F$ étant régulièrement fermé par rapport à $\overline{E}$ l'est également par rapport à $F=\left(\frac{\overline{E}}{\overline{G}}\right)$.

Donc, d'après un résultat de M. Banach* $\frac{E}{G}$ et par suite F sont des espaces réguliers. D'où il suit que la sphère unitaire de F est faiblement compacte [1] (au sens de la convergence faible des éléments). La construction même de F montre que la sphère unitaire de $\overline{E}$ est faiblement compacte. Mais alors en vertu du théorème de V. Gantmakher et de l'auteur [1] il en est de même pour la sphère unitaire de E.

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* Car, si tout sous-espace fermé $F \subset \overline{E}$ est régulièrement fermé, l'espace E est régulier [4].

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PHYSICS

NON-STATIONARY FREE CONVECTION

By D. A. FRANK-KAMENECKIJ

(Communicated by N. N. Semenov, Member of the Academy, 15. I. 1938)

All the existing theoretical work concerning free convection deals with a stationary regime. The question of non-stationary free convection, so far as the author is aware, has not been discussed.

The non-stationary free convection process is described by two equations: the heat conductivity equation:

$$\frac{\partial \vartheta}{\partial t} + (v \operatorname{grad}) \vartheta = \varkappa \Delta \vartheta \tag{1}$$

and the hydrodynamic equation, which in assumptions usually accepted in the theory of free convection can be written as:

$$\frac{\partial v}{\partial t} + (v \operatorname{grad}) v = g\beta\vartheta + \nu \Delta v. \tag{2}$$

Here ϑ is the temperature difference, $\varkappa$ — thermal diffusivity, ν — the kinematic viscosity, β — the coefficient of thermal expansion (in the case of ideal gases $\beta = \frac{1}{T}$).

Applying dimensional analysis to these equations, we arrive at the following set of non-dimensional parameters (criteria of similarity):

$$\frac{v_0 \tau}{d}; \quad \frac{\varkappa \tau}{d^2}; \quad \frac{g\beta\vartheta_0 \tau}{v_0}; \quad \frac{\nu\tau}{d^2}. \tag{3}$$

Here d is the linear dimension of the system under consideration, τ — characteristic time, v_0 and ϑ_0 — characteristic velocity and temperature difference, i. e. the values of these quantities relating to a certain moment and definite points, or averaged according to a definite law.

Without entering into detailed analysis of initial and boundary conditions (conditions of uniformity), we may note merely that free convection is formulated as a process where these conditions do not contain the velocity; consequently, the determining parameters must be formed by such a combination of parameters (3) which eliminates the velocity v_0. Multiplying the first and third parameters we obtain the determining criterion:

$$\frac{g\beta\vartheta_0 \tau^2}{d}.$$

Dividing the third parameter by the first we obtain the characteristic velocity:

$$v_0 = \sqrt{gd\beta\vartheta_0}.$$

The second and fourth parameters for a given substance are distinguished only by a constant factor $\frac{\varkappa}{\nu} = Pr$, known in the theory of similarity under the name of Prandtl's criterion. Hence, it is more rational to use one of them (e. g. the second) and the quantity Pr, which changes its value only when we pass from one substance to another.

Finally we obtain for non-stationary free convection the following sequense of determining non-dimensional parameters (criteria):

$$\frac{g\beta\vartheta_0\tau^2}{d};\ \frac{\varkappa\tau}{d^2};\ \frac{\varkappa}{\nu}. \tag{4}$$

The criterion of the similarity of fields of physical constants $\frac{\Delta T}{T}$ is of no importance in the present connection.

The solution of system (1), (2) will give the spatial distribution of the temperature and velocity as a function of the time. This solution contains the time in the form of two independent non-dimensional variables $\frac{t}{\tau_1}$ and $\frac{t}{\tau_2}$, where τ_1 and τ_2 are two characteristic times, determined by the first and second parameters of the sequence (4):

$$\tau_1 = \frac{d^2}{\varkappa} \tag{5}$$

$$\tau_2 = \sqrt{\frac{d}{g\beta\vartheta_0}} \tag{6}$$

τ_1 is the usual characteristic time of heat transfer by pure conduction; the quantity τ_2 is essentially hydrodynamic in character and is identical (omitting a constant numerical factor) with the time required by a free falling body for passage through the distance d, if the density of the body differs from that of the surrounding medium by the quantity $\beta\vartheta_0$ («Umlaufzeit» of German authors).

The solution of system (1), (2) is consequently of the form:

$$\frac{\vartheta}{\vartheta_0} = f\left(\frac{x}{d};\ \frac{t}{\tau_1};\ \frac{t}{\tau_2}\right), \tag{7}$$

$$\frac{v_0}{\sqrt{gd\beta\vartheta_0}} = f\left(\frac{x}{d};\ \frac{t}{\tau_1};\ \frac{t}{\tau_2}\right). \tag{7a}$$

Correspondingly, the coefficient of heat transfer can be obtained as $\alpha = \frac{\lambda}{\vartheta_0}\left(\frac{\partial\vartheta}{\partial x}\right)_{x_0}$. Here x denotes the space coordinates, λ — thermal conductivity; the derivative $\left(\frac{\partial\vartheta}{\partial x}\right)_{x_0}$ is taken in the direction perpendicular to the boundary surface of the system; its value is taken for $x = x_0$ which is on this surface. Introducing the non-dimensional variables $\theta = \frac{\vartheta}{\vartheta_0}$ and $\xi = \frac{x}{d}$, we can rewrite (7) thus:

$$\theta = f\left(\xi;\ \frac{t}{\tau_1};\ \frac{t}{\tau_2}\right) \tag{8}$$

and the coefficient of heat transfer will take the form $\alpha = \frac{\lambda}{d}\left(\frac{\partial\theta}{\partial\xi}\right)_{\xi_0}$, or

$$\frac{\alpha d}{\lambda} = \left(\frac{\partial\theta}{\partial\xi}\right)_{\xi_0}. \tag{9}$$

The quantity $\frac{\alpha d}{\lambda} = Nu$ is the non-dimensional coefficient of heat transfer, usually called Nusselt's criterion. The value of $\left(\frac{\partial \theta}{\partial \xi}\right)_{\xi_0}$ can be obtained by differentiating (8) and putting $\xi = \xi_0$ into the result.

We finally obtain for the coefficient of heat transfer:

$$Nu = \frac{\alpha d}{\lambda} = f'\left(\xi_0; \frac{t}{\tau_1}; \frac{t}{\tau_2}\right), \tag{10}$$

whence the quantity ξ_0 gives the point on the boundary surface where the coefficient of heat transfer is determined. For similar points of similar systems it follows:

$$Nu = f'\left(\frac{t}{\tau_1}; \frac{t}{\tau_2}\right). \tag{11}$$

We can evidently write the solution of system (1), (2) with only one non-dimensional time — τ_1 or τ_2, but thence one more non-dimensional parameter equal to the ratio of these two quantities $\frac{\tau_1}{\tau_2}$ must appear in the solution, and the formulae (8) and (11) take the form:

$$\theta = f\left(\xi; \frac{t}{\tau}; \frac{\tau_1}{\tau_2}\right), \tag{12}$$

$$Nu = f'\left(\frac{t}{\tau}; \frac{\tau_1}{\tau_2}\right), \tag{13}$$

whence for τ we can arbitrarily choose each of the quantities τ_1 or τ_2.

What then is the quantity $\frac{\tau_1}{\tau_2}$? Putting the values of τ_1 and τ_2 from (5) and (6) we obtain:

$$\frac{\tau_1}{\tau_2} = \sqrt{\frac{g d^3}{\varkappa^2} \beta \vartheta_0} = \sqrt{Gr} \cdot Pr \tag{14}$$

for $\frac{g d^3}{\varkappa^2} \beta \vartheta_0$ is the non-dimensional parameter known in the theory of stationary free convection as Grasshof's criterion (Gr).

We can now write (12) and (13) as follows:

$$\theta = f\left(\xi; \frac{t}{\tau}; Gr\right), \tag{15}$$

$$Nu = f'\left(\frac{t}{\tau}; Gr\right), \tag{16}$$

which is a natural consequence of the theory of similarity and could be easily obtained directly.

The formulae (8) and (9) contain however more than the corresponding equations (15) and (16). From (11) and (12) it follows that in the limiting case when the value of one of the quantities τ_1 or τ_2 is very large (assuming that such a limit exists), the phenomenon will be completely described by means of the remaining quantity.

If $\tau_2 \gg \tau_1$, i. e. $Gr \ll 1$, we obtain the limiting case of pure conduction, whence the rôle of the relaxation time is played by τ_1. It is physically evident that this limiting case always exists, i. e. that when Gr approaches zero the functions f and f' always tend to certain limits.

If $\tau_1 \gg \tau_2$, i. e. $Gr \gg 1$, one can obtain the other limiting case — that of perfect turbulent motion, which corresponds to the limiting case discussed previously by the author [1] in the theory of stationary free convection.

In this case the characteristic time of heat transfer must be proportional to the quantity τ_2, i. e. directly proportional to the square root of the linear dimension of the system and inversely proportional to the square root of temperature difference. The latter relationships have already been found in the paper cited from a discussion of the quasi-stationary case.

It is necessary to point out the essential difference between the two limiting cases considered. If the existence of the first (absence of convection) is physically evident, then the second may not necessarily occur.

In each actual case it may happen that when *Gr* tends to infinity, the functions f and f' do not tend to a definite limit, and that the second limiting case arises, e. g. only when the coefficient of heat transfer itself becomes infinitesimal.

On the other hand, although the non-dimensional factor representing the ratio between τ_1 and the exact relaxation time of heat transfer by pure conduction is usually of the order of unity, this is not generally true for the corresponding factor for the second limiting case. On the contrary, discussion of simple examples leads to the conclusion that the characteristic time of heat transfer for pure convection, although proportional to τ_2, may yet differ notably from it numerically.

Comparing the conclusions of the present paper with those of the preceding one [1], we conclude that Grasshof's criterion is actually a quadratic quantity and that it would be more rational to use, as a determining criterion in the theory of free convection, the quantity:

$$\frac{\tau_1}{\tau_2}\sqrt{\frac{gd^3}{\varkappa^2}\beta\vartheta_0} = Gr^{\frac{1}{2}}Pr.$$

Conclusions

1. The temperature distribution and the coefficient of heat transfer for non-stationary free convection can be expressed in the form of functions of two non-dimensional times $\frac{t}{\tau_1}$ and $\frac{t}{\tau_2}$ where $\tau_1 = \frac{d_2}{\varkappa}$ is the characteristic time of heat transfer by pure conduction and $\tau^2 = \sqrt{\frac{d}{g\beta\vartheta_0}}$ is hydrodynamical in character («Umlaufzeit»).

2. Grasshof's criterion (or more exactly $GrPr^2$) is equal to the square of the ratio of the quantities τ_1 and τ_2; consequently the functions quoted can be transformed in such a way that only one of the arbitrarily chosen quantities τ_1 or τ_2 remains together with Grasshof's criterion.

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PHYSICS

THE TEMPERATURE DISTRIBUTION IN A REACTION VESSEL AND THE STATIONARY THEORY OF THERMAL EXPLOSIONS

By D. A. FRANK-KAMENECKIJ

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The theory of thermal explosions advanced by Semenov (1) and Todes (2) has been developed on the assumption of equal temperatures at every point of the vessel. In other words the theory dealt only with the average temperature of the gas mixture. The results contain therefore the average coefficient of heat transfer, which is a quantity essentially indeterminate in character and is actually dependent not only on the composition of the gas mixture and on the form and dimensions of the vessel but also on the distribution of temperatures and gas convection velocities in the vessel.

Dimensional analysis of the equations of heat conduction and hydrodynamics and their boundary conditions shows that in the stationary theory, i. e. neglecting the change of concentration of the reacting substances during the induction period, and neglecting also the temperature gradient in the walls, which is permissible in all actual cases, the critical condition of inflammation consists in a relation among the non-dimensional parameters:

$$\delta = \frac{E}{RT_0^2}\,\frac{Q}{\lambda}\,r^2 W_0 = \frac{E}{RT_0^2}\,\frac{Q}{\lambda}\,r^2 z e^{-\frac{E}{RT_0}},$$

$$u_0 = \frac{RT_0}{E},$$

$$Gr = \frac{gr^3}{\varkappa^2}\,\frac{RT_0}{E},$$

where $W_0 = z e^{-\frac{E}{RT_0}}$ is the reaction velocity at the wall temperature T_0, Q—the heat of reaction, E—the activation energy, λ—the thermal conductivity and $\varkappa$—the thermal diffusivity of the gas mixture, r—the radius of the vessel. The quantity Gr is analogous to a parameter known in the theory of free convection under the name of Grasshof's criterion.

We have shown that when u_0 approaches zero, the relation quoted tends to a limiting relation between δ and Gr. Since in practice we always have $RT_0 \ll E$, we can restrict the discussion to this limiting case. It corresponds physically to low pre-explosion elevation of temperature, for the maximum $\frac{\Delta T}{T}$ before the explosion is of the order of $\frac{RT_0}{E}$.

At low pressures and small vessel dimensions $Gr \to 0$ and we have the limiting case of heat transfer by pure conduction where the influence of convection is negligibly small. This can be completely solved analytically.

In this case the only differential equation is the equation of heat conduction in a reacting gas mixture which in non-dimensional variables $\theta = \frac{E}{RT_0^2}(T - T_0)$ and $\xi = \frac{x}{r}$ can be written in the form:

$$\Delta_\xi \theta = -\delta e^\theta,$$

with boundary conditions: $\theta = 0$ at $\xi = 1$, $\frac{d\theta}{d\xi} = 0$ at $\xi = 0$ for symmetrical vessels.

Here the quantity $\frac{E}{RT}$, which stands in the exponent of Arrhenius's law, is expanded in a series of powers of $\frac{T-T_0}{T_0}$ and powers higher than the first are neglected, which corresponds to the assumption $u_0 \ll 1$.

The solution of this equation gives the stationary distribution of temperature in the reacting mixture as a function $\theta = f(\xi, \delta)$ depending on one parameter δ. The critical condition of inflammation is the condition of the impossibility of such a stationary regime which can contain only this parameter δ and is, consequently, reduced to a constant critical value of δ for geometrically similar vessels.

We have calculated these critical values of δ: for an infinite plane-parallel vessel $\delta = 0.88$; for a cylindrical one $\delta = 2.00$; for a spherical one $\delta = 3.32$. For the plane-parallel vessel the equation can be integrated in elementary functions. For the other cases quoted the result is obtained by means of numerical integration.

Calculations based on experimental data of Rice [3], Appin [4], and Yakovlev show that the usual conditions of the experiment lay in the region very close to that of pure conduction. Experimental critical values of δ satisfactorily agree with those given above.

The opinion of Rice and Todes that under usual experimental conditions the heat loss from the vessel is incomparably larger than that calculated for pure conduction, is seen to be completely wrong. Actually the rôle of convection can be important only at higher pressures.

The dependence between the coefficient of heat transfer and the diameter of the vessel, found by Rice to be inversely proportional, also corresponds exactly to the laws of pure conduction.

I take this opportunity of thanking E. E. Fridman who carried out the numerical calculations.

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PHYSICS

ENERGY LOSS OF FAST ELECTRONS BY RADIATION

By L. A. ARCIMOVIČ and V. A. CHRAMOV

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The study of γ-radiation, emitted by electrons in the nucleus field, is nowadays of great importance, since it will give clear understanding of processes involved in the interactions of fast electrons with nuclei, as well as of phenomena we met with in studying the cosmic rays.

The theory of energy loss of fast electrons by radiation for a relativistic case (the velocity of electrons being near the velocity of light) was worked out by Bethe and Heitler. According to this theory the energy radiated by an electron in an elementary act of scattering is approximately proportional to the square of the atomic number of the element, and rapidly increases with the increase of electron energy.

All experimental data on this question obtained up to the present time, refer to electrons of energies less than 300—400 kV. Electrons of such small energies do not show their relativistic properties clear enough, and in this case the radiative energy loss amounts only to some tenth fractions of p. c. in the total balance of energy lost by electrons in passing through the substance.

In the present paper the authors have tried to make a quantitative investigation of γ-radiation, emitted by scattered electrons of energies 600—3000 kV, for various substances. As a source of fast electrons radioactive substances (radon) were used, whose intensity in different experiments varied from 50 to 500 millicurie. For isolation of a monochromatic beam of electrons we used a vacuum magnetic spectrograph with focussing by a longitudinal field. For focussing the field was used, produced by a short solenoid (magnetic lens). The energy interval within which the radiation emitted by scattered electrons was studied, made 5% of the energy of electrons focussed at the outlet window of the apparatus.

On the path of an isolated beam of electrons we placed a target made of the substance in which the stopping of electrons was studied. The thickness of the target was equal to the mean free path of electrons in a given substance. Thus the energy radiated by an electron with a given initial velocity on the whole length of its path has been measured in our experiment. We have studied the dependence of the intensity of this radiation on Z (the atomic number of the element) and on the initial energy of electrons, and have measured the absolute value of the output of γ-ray energy.

For the radiation intensity measurements a Geiger counter and ionization chamber were made use of. The counter was used only in measuring

the dependence of the radiation intensity on Z. The initial energy of electrons was 2200 kV. Radiation emitted by electrons passing through targets of C, Al, Cu, Sn and Pb was measured. The counter was placed 5 cm from the target at an angle of 50° to the direction of the electron beam hitting up on the target. The results of our measurements (with correction for the absorption of γ-rays in targets) are shown in Fig. 1a. It may easily be seen that the intensity of radiation emitted by the electrons proves to be nearly proportional to Z. The Fig. 1b shows the data of analogous measurements, carried out with an ionization chamber. Special interest is presented by the study of cases when an electron losses by radiation a considerable amount of its energy, i. e. when the energy of the emitted γ-quantum

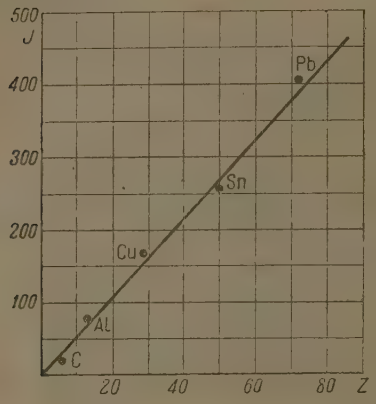

Fig. 1a. J—intensity.
Z—atomic number.

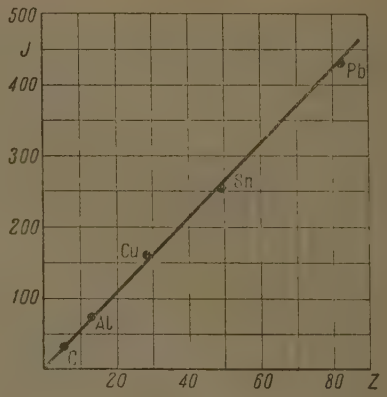

Fig. 1b. J—intensity.
Z—atomic number.

constitutes a considerable part of the energy of the electron (a case of strong interaction).

In order to study the output of such hard γ-quanta in dependence on Z, we have filtered the γ-radiation through 7 mm of lead. The results of these measurements, alro carried out by means of a counter, show the same linear dependence of the intensity on Z.

The results of all experiments mentioned are in good agreement with the theory since the dependence $J \sim Z^2$ for an elementary act of interaction is equivalent to the dependence $J \sim Z$ valid for the case when electrons stopped in the substance entirely lose their velocity.

To measure the dependence of radiation emitted by hindered electrons on their energy, an ionization chamber was used, placed immediately behind the target. Thus the total intensity of γ-radiation emitted forward at an angle equal to 2π could be measured in the chamber. The radiation emitted by the scattered electrons was to be measured on an intensive background of scattered γ-rays emitted by the radioactive substance itself. Therefore we had recourse to the zero method, based on the application of the compensating chamber. The ionization chamber was connected with a string electrometer of sensibility 200—300 scale divis./volts. The ionization current originated by γ-rays emitted on the stopping of electrons of various energies from 675 kV to 2940 kV in the target of Cu was measured. The corrections for the absorption of γ-rays in the target were made.

The results of measurements of the dependence of the radiation intensity on the energy of electrons are shown in Fig. 2.

The energy of electrons in kilovolts has been plotted on the axis of abscissae, the intensity of the emitted radiation per electron on the axis of ordinates. In the same figure there is shown (full line) the curve of the dependence of J on the energy of electrons, calculated from the formulae, derived in Bethe and Heitler's theory.

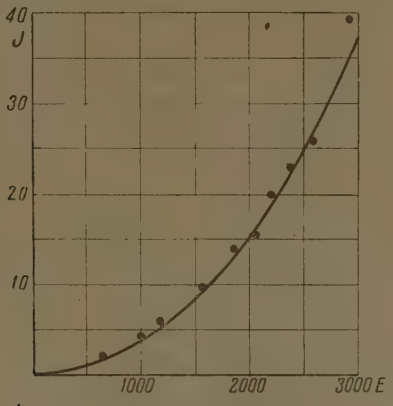

Fig. 2. J—intensity.
E—electron energy.

We may consider adequate the agreement of the computed and experimental data, especially if we take into account that Bethe and Heitler's theory is an approximate one, and the experimental errors are rather large because of the small intensity of the phenomenon observed.

Having calibrated the ionization chamber for γ-rays by means of etalon of radium we have also been able to determine the absolute yield of γ-radiation.

Total radiation energy loss of electrons of energy 2200 kV, scattered in Cu was found equal to $2.8 \pm 0.6\%$ of the initial electron energy. After Bethe and Heitler's theory this value should be $\sim 2.9\%$. Thus in this point the experimental data are also in satisfactory agreement with theory.

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PHYSICS

THE PROBABILITY OF CREATION OF PAIRS IN KRYPTON BY γ - RAYS

By L. V. GROŠEV and I. M. FRANK

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1. The most direct method of studying pairs is to observe them in a Wilson cloud chamber. Since the probability of pair formation in gases is very small, the usual proceeding is to observe the formation of pairs in metallic plates(1) placed within the chamber. This produces a marked increase of pair yield, as compared to that of heavy gas, provided the plate is sufficiently thick. Such a plate, however, causes considerable distortions, due to the scattering and absorption of electrons and positrons within it. These difficulties may be avoided by observing the pair formation in the very gas filling the Wilson cloud chamber. These conditions afford the most direct means of determining the effective cross-section for the creation of pairs, and of inquiring into their distribution in space.

In the experiment reported here the creation in krypton of pairs due to γ-rays ThC″ was observed. When we were setting to work only a few pictures of pairs in gas were available [F. Joliot(2) and D. Skobelzyn(3)]. Later on pictures of pairs in krypton were obtained by Immelmann (4) and then by Bothe and Klarmann(5) (4 pairs in Kr and 9 in Xe). Recently rather extensive data on pairs in argon and iodine were gained by Simons and Zuber(6).

A special hermetic Wilson cloud chamber was built for the purpose. Previous to filling the chamber with krypton air was drawn off by means of an oil pump, and the chamber was tested for vacuum for a long time. The experiment was carried out with alcohol vapour. An analysis of the gas, made after the close of the experiment (two months after the filling of the chamber), showed its composition without any change. This is confirmed too by the fact that two series of pictures taken one after the other with a month's interval show the same result in regard both to Compton electrons and the pairs.

The chamber was placed in the magnetic field of a solenoid. The density of the field for the first series of photographs was 1500 gausses on the average, and for the second, 1250 gausses. The photographs were taken on a continuous film by means of a stereoscopic camera. The aperture ratio of the objectives was 1 : 1.8; the focus, 35 mm.

A 10 mg Ra equivalent of RdTh served as a source of γ-rays. It was placed in a lead block, 45 cm from the center of the chamber. The thickness of the lead block in the direction source—chamber was 27 cm;

in other directions, 13 cm. The γ-rays filtered through 3 mm of lead entered the chamber through a narrow bore (3 mm in diameter) made in the lead block. The diameter of the beam of γ-rays was not more than 1 cm throughout their path in the chamber. With a chamber filled with krypton, 2200 stereoscopic pictures were obtained.

2. The following difficulty encumbers the observation of pairs in the Wilson chamber. Apart from the pairs a number of electrons scattered in the gas at large angles («reflection electrons») is always observed in the chamber. In two cases only we may be sure of a formation of a pair rather than of a «recoil electron»: 1) If the electron is sure to possess a higher energy than the positron. Such a pair may be interpreted only as a non-elastic scattering of the positron, which is hardly probable. 2) If the direction of the positron of the pair is fixed in some way, e. g. by an abrupt energy loss on its motion, or by the formation of a fast δ-electron. The number of those pairs is, however, small. In the pictures obtained, they make up only 1/10 of the total number of pairs.

In our experiment we used a narrow well defined beam of γ-rays. Thus we reduced the region where the vertices of the pairs might be found. Besides, with a smaller diameter of the beam it became less probable that a Compton electron created in the beam should there undergo a reflection. Further the electrons and positrons of the pairs have, as a rule, their track ends lying outside the beam if the latter is narrow. Consequently, the probability diminishes of a pair being mistaken for a Compton electron reflected within the beam. In most cases this allows the pairs to be safely distinguished from the Compton electrons.

Apart from the particles, produced within the beam, a background of electrons formed by scattered γ-rays and radioactive impurities is always present in the chamber. For these electrons the ratio of incidental reflections within the beam to the number of pairs does not decrease with the narrowing of the beam, inasmuch as the probability both of the reflections and of pair formation is proportional to the volume of the beam. The probable number of such reflections may be determined from the distribution of the reflections in the regions directly adjacent to the beam. Therefore we investigated the statistical distribution of the reflection and pair vertices in the plane of the chamber, making no difference between the two.

All the pairs and reflections have been subdivided, with reference to their energies, into two groups*.

To the first group pairs and reflections were referred for which the total energy of both branches lay within 500—1000 ekV (Fig. 1a); and to the second group, those with an energy over 1000 ekV (Fig. 1b). In Figs 1a and 1b the number of reflection and pair vertices is given, falling within the parallel bands of equal width (11 mm), into which the plane of the Wilson chamber was divided for calculation. The zero band corresponds to that part of the chamber where the beam of γ-rays passes through.

As may be seen from a comparison of Figs 1a and 1b, the distribution is quite different within two energy intervals. For the first interval of energy (Fig. 1a) the zero band is not to be distinguished from the others. Consequently, the total number of pairs in the beam for those energies is so small that

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* Since the probability of electron scattering at small angles is very great, the reflections at angles less than 45° were not taken into account. Thus the pairs with an angle φ over 135° did not enter into the calculation which was made without reference to the vertices being due to reflections or to pairs. However, an analysis of the angular distribution shows that there is a very small probability of formation of pairs with an angle φ over 135°, if any.

it cannot be distinguished with certainty. For energies over 1000 ekV, the number of reflections outside the beam has markedly diminished, whereas in the zero band coinciding with the beam a strong maximum is seen. It gives the total number of pairs and reflections in the beam, which is equal to 80. The number of reflections in the beam may be determined by means of a comparison with the adjacent bands. Fig. 1b shows that in our case their number may vary from 2 to 4.

In our diagrams, the reflections from the beam are omitted if the origin of Compton electrons to which they are due is seen. The distribution of reflections for these electrons is shown in Fig. 1b by a dotted line.

3. The main portion of pairs in the case of ThC″ radiation is due to photons with an energy $h\nu$ 2620 ekV. Therefore, it would be desirable to isolate the pairs, corresponding to photons of this energy only. For krypton, however, this is impossible, for the intense scattering of electrons prevents an accurate measurement of energy from the curvature of tracks. Moreover, the subdivision of pairs and reflections into two groups after their energies is rather arbitrary.

Certain number of pairs from the line $h\nu$=2620 ekV may possibly fall into the first interval (energies below 1000 ekV). The number of such pairs, however, cannot be large, as it is shown in Fig. 2, where distribution according to energies is given for the pairs obtained.

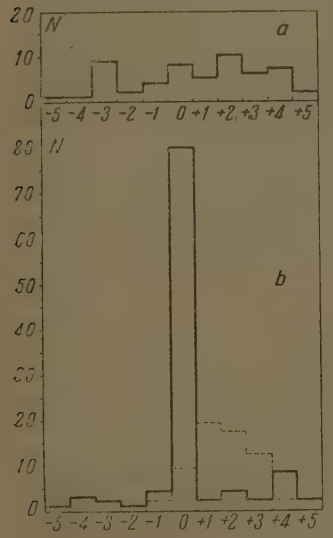

Fig. 1a, 1b.

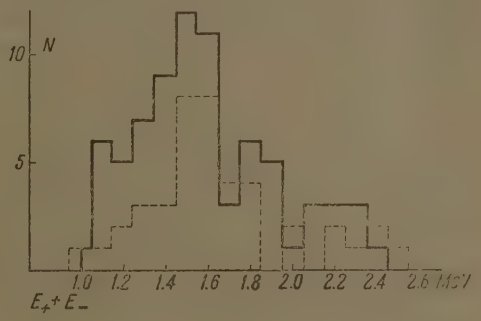

Fig. 2.

In Fig. 2 instead of the strong maximum, corresponding to the energy of 1600 ekV, a rather broad curve is seen*. (In Fig. 2 the dotted line shows analogous results, obtained previously by Simons and Zuber with argon and iodine.) Computing for all the pairs their average energy, we obtain the value of 1580 ekV. This shows that the principal line in the formation of pairs is actually the line 2620 ekV. Apart from it the weak lines 2200 and 3200 ekV may be also of certain importance [according to Alikhanov(7), the number of pairs, corresponding to these lines, must be of the order of 10%].

4. The results obtained make it possible to calculate directly the effective cross-section for the process of pair formation. For this purpose it is sufficient to determine the ratio of the number of pairs to the number of

* Bothe and Klarmann(5) in their work counted only those pairs the energy of which did lie within the interval from 1400 to 1800 ekV. In Fig. 2 it is seen that a considerable per cent of pairs from the line 2620 ekV was lost in this selection.

Compton electrons, produced by the same γ-ray line. The total number of the Compton electrons was calculated as follows. From the measurements of the curvature, fast electrons were selected from the beam, the energy of which was over 1660 ekV (for the principal line of radiation ThC″ this corresponds to the electrons ejected at an angle θ less than 20°). Electrons of such energies may be produced besides the principal line also by photons corresponding the lines 2200 and 3200 ekV. Their number, however, is not very large (less than 10 per cent). For that reason all the electrons with energies above 1660 ekV, shown in our photographs, have been referred to the principal line, and from their number the total number of electrons due to this line has been determined after Klein-Nishina's formula. The value 1450 has been obtained. Since the energy measurements are inaccurate some error may be made in selecting electrons. Therefore the same calculations were made for energies over 2160 ekV (θ less than 10 per cent for the principal line). The result of both calculations was the same within several per cent.

The total number of pairs in our photographs, less possible reflections, is 77. Hence, from the ratio of the number of pairs to the number of electrons, we find the effective cross-section for the formation of the pairs $\varkappa = 0.25 \cdot 10^{-24}$ cm^2 (the effective cross-section for the Compton effect in Kr calculated per atom is assumed to be equal to $\sigma = 4.43 \cdot 10^{-24}$ cm^2). As said above in all the calculations the spectrum of radiation ThC″ was arbitrarily taken as monochromatic. The result does not change essentially if the presence of other γ-lines in the spectrum of TC″ is taken into account.

Theoretically the effective cross-section for the formation of pairs[8] in krypton is $\varkappa = 0.48 \cdot 10^{-24}$ cm^2 in Born's approximation. We assume that in the second approximation the dependence of the effective cross-section for krypton upon the atom number will be expressed by the equation[9]

$$\varkappa = a\left(\frac{Z}{137}\right)^2 + b\left(\frac{Z}{137}\right)^4,$$

the first member being $\varkappa$ in Borns's approximation. Hence after having determined the coefficient b from the value $\varkappa$, calculated for lead by Hulme and Jaeger[10], we find for krypton $\varkappa = 0.50 \cdot 10^{-24}$ cm^2. Thus the experimental effective cross-section is about half as large as the theoretical one.

This discrepancy, however, is of no real importance, since the experimentally established $\varkappa$ has been understated.

Therefore it should be held that the effective section found does not disagree with the theory.

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PHYSICAL CHEMISTRY

THE MECHANICAL PROPERTIES OF FILMS AND THE STABILITY OF FOAMS

By P. A. REHBINDER, Corresponding Member of the Academy, and A. A. TRAPEZNIKOV

In connection with our investigations into the rôle played by the mechanical properties of adsorption layers(1) in the stability of films, individual bubbles and foam (see the next communication*), i. e. in the stabilizing action, we undertook some measurements of surface viscosity in solutions of a typical foam-forming agent—sodium oleate. In aqueous solutions of saponine which yields still more stable foam, the high resistance to shear exhibited by the surface adsorption layer constituting elastico-brittle «film jelly» may be readily measured**; the same measurements for pure sodium oleate are hampered by the comparatively low values of surface viscosity.

Schütt(2) attempted to measure the surface viscosity in solutions of Venetian soap with glycerine and failed to detect it, probably in view of the insufficient sensitivity of the apparatus used. The appearance of surface viscosity in the course of time was ascribed by him to the effect of carbon dioxide. This gradual appearance of the increased surface viscosity of the surface layer, however, could also be attributed to the colloidal ageing of sodium oleate solution both in the bulk and in the adsorption layer, i. e. to the formation of micelles in solution, due to increased concentration. To eliminate the effect produced on solutions by CO_2 and thus ensure the possibility of extending the time of measurements, we designed an apparatus which enabled the measurements to be conducted in an atmosphere of either nitrogen or argon at a constant temperature. The measurements carried out under such conditions, using a horizontal disc oscillating about a vertical axis and half immersed in the solution [i. e. our usual method(1)], failed to reveal any surface viscosity, even after a lapse of some days of ageing of solutions, and thus substantiated the assumption advanced by Schütt, according to which any surface viscosity, that can be measured under such conditions may only be due to the chemical action of atmospheric CO_2. Moreover, these measurements showed that the viscosity of sodium oleate surface layers undergoes no appreciable rise on ageing. The negative result of the measurements, obtained under the above conditions, is

* P. 427 C. R. Acad. Sci. URSS, XVIII, № 7.

** In measuring the mechanical strength of adsorption layers in aqueous solutions of saponine we succeeded in obtaining complete stress-strain curves up to the strain corresponding to rupture.

associated with the fact that the damping of the oscillations of a disc immersed in the surface layer of a solution is wholly due to the bulk properties of the solution surrounding the disc, the surface viscosity being comparatively low. To reduce the effect of the bulk layers, which disguised the mechanical properties peculiar to the surface layer, we elaborated a method of measuring the viscosity in two sided aqueous solution films, formed between two concentric horizontal rings made of platinum wire (d=1.0 mm); the outer ring (6.12 cm in diameter) was rigidly fixed, whilst the inner one (5.39 cm in diameter), replacing the disc, was suspended on a tungsten torsion wire and was caused to oscillate in the plane of the film. The film between the rings was formed by lowering the level of the solution, contained in a cylindrical glass cup, by means of a tube sealed in the bottom to prevent the formation of foam. The logarithmic decrement of oscillations, λ, which to a first approximation is proportional to viscosity, was taken as a characteristic of the mechanical properties of the films. The films of th solutions investigated, stretched between the rings of the oscillating sy

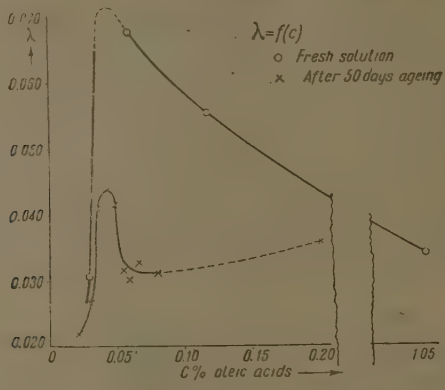

Fig. 1.

stem, were altogether analogous to the films constituting the foams of the respective solutions. Using these stretched films we obtained a viscosity isotherm, $\lambda = f(c)$, for films of sodium oleate solutions of various concentrations c (Fig. 1).

Sodium oleate was prepared, as is customary in our laboratory[3], by saponification of pure oleic acid with pure sodium hydrozide in strictly stoichiometric proportions in an aqueous medium over a boiling water bath in absence of CO_2. The measurements were carried out after a lapse of 2 days since the preparation of the initial solution (c=1.05%), which was subsequently diluted to obtain solutions of lower concentrations. The solutions were subjected to measurements successively from lower to higher concentrations, so as to eliminate any appreciable effect of the ageing of solutions during the measurements upon the form of the curves $\lambda = f(c)$. The time which elapsed between the measurements of solutions of extreme concentrations (0.03 and 1.05%) amounted to four days (for the top curve in Fig. 1). Similar measurements, carried out after the initial solution (c=1.05%) had undergone colloidal ageing for 50 days, also showed that the curve $\lambda = f(c)$ possesses a maximum (Fig. 1) which in both cases corresponds to the saturation of the adsorption layer, i. e. to c=0.045 — 0.055%.

The general lowering of the position of the curve λ as well as some alteration of its form observed after 50 days may be ascribed to the long period of ageing of the solution. The interval of time between the measurements of 0.025 and 0.06% solutions did not exceed 2 days.

It should be noted that, owing to the high surface activity of sodium oleate, such viscosity measurements may be somewhat affected, especially in the case of an imperfectly centered system, by the local surface tension variations which are liable to enhance the damping of oscillations [apparent viscosity or the Marangoni effect(4)].

This effect was practically absent in our measurements on the viscosity of insoluble unimolecular films. For two-sided sodium oleate films, it is apparently also very slight. The latter statement is vindicated by the independence of the logarithmic decrement λ on the amplitude of oscillations within fairly wide limits. The isotherm viscosity for films of sodium oleate aqueous solutions $\lambda = f(c)$, is quite similar to the surface viscosity curve, $\lambda = f(\Gamma)$, obtained for adsorption films of cetyl alcohol at various

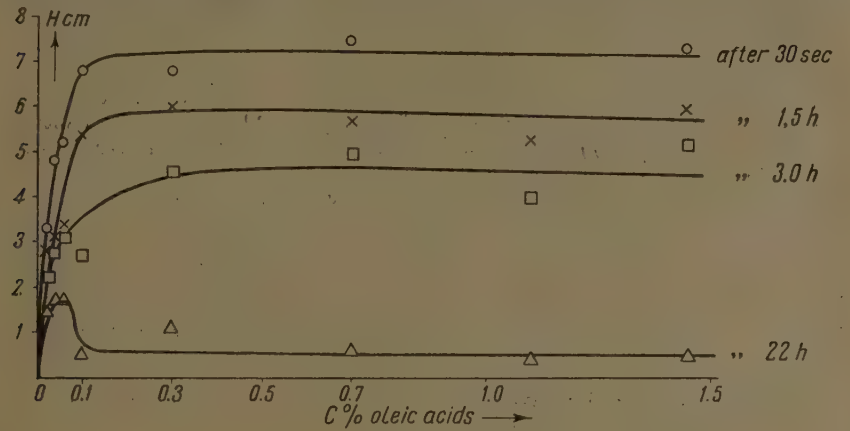

Fig. 2.

surface concentrations. The maximum on the isotherm $\lambda = f(c)$ and the subsequent decrease in the viscosity of the films with increase in concentration are apparently associated with the breaking of homogeneity in the adsorption layer as well as local coagulation due to the rise in concentration in the bulk phase between the adsorption layers.

The values of the logarithmic decrement λ, obtained for several films of one and the same solution, are in good agreement with each other.

Simultaneously with the viscosity measurements in films of sodium oleate solutions, we measured the stability of foam as a function of concentration in the same solutions. The foam was formed by shaking a definite volume of solution in a glass cylinder in a standard way. The height, i. e. the volume of the foam column, was determined as a function of the life of foam. This enabled the time of coalescence (destruction) of a definite column of foam H, i. e. the stability of foam(3), to be determined. The initial (maximum) volume of foam in sodium oleate solutions continuously rises with concentration, reaching a constant maximum value at about $c = 0.1\%$ (retained up to 1.5%). In measuring the volumes of foam after a sufficient lapse of time since its formation, a clearly expressed maximum of the foam volume was observed on the isochronous isotherm $H = \varphi(c)$, between 3 and 22 hours

(Fig. 2). The longer the time of measurement, the lower does the whole isotherm lie. As the time of measurement is increased the maximum of the foam volume becomes sharper and shifts in the direction of lower concentrations corresponding for 22 hours, to the region of saturation of the adsorption layer, at about 0.04%. This shows that the stability of foam, obtained for sufficiently long periods of the existence of a foam column, exhibits a sharp maximum when plotted against the concentration, and possesses a form similar to that of the viscosity isotherms of films.

It is possible that on observing the coalescence of foam columns for a sufficiently long time, the maximum of foam stability will be found to shift towards still lower concentrations, i. e. will take place before the saturation of the adsorption layer, or, in other words, before the maximum of surface viscosity, is reached. This is exactly what was found for the foam obtained from aqueous solutions of saponine. At high concentrations the spontaneous destruction of a foam column also occurs with a high velocity. For this reason a maximum appears on the isotherms of foam formation, $H=\varphi(c)_\tau$ according as τ is increased. This maximum gradually shifts (for 3 to 10 days) towards the lower concentrations smaller than those corresponding to saturation of the adsorption layer, i. e. $c=0.15\%$. The kinetics of foam destruction are different for various heights of the column. The collapse of films is more rapid in the top layers of foam. The films of the lower bubbles are thicker and exist for longer periods owing to the constant supply of solution trickling down from the top layers: the lower bubbles exist for a longer time at the expense of the upper ones. The process of bubble collapse which determines the position of the maximum of foam stability in unsaturated adsorption layers, is associated with a peculiar phenomenon of local syneresis in adsorption layers*, occuring on decrease in the thickness of the bubble films, i. e. on trickling of the liquid down the film. The rapid decrease in the thickness of the film, and the eventual syneresis, are opposed by the hydration of molecules and micelles in the adsorption layer.

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* For details on this subject see the next communication. p. 427, C. R. Acad. Sci. URSS, XVIII, № 7.

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PHYSICAL CHEMISTRY

STABILIZING EFFECT OF ADSORPTION LAYERS AND THEIR MECHANICAL PROPERTIES

By A. A. TRAPEZNIKOV and P. A. REHBINDER, Corresponding Member of the Academy

Simultaneously with a study of the mechanical properties of adsorption layers as functions of the surface concentration(1), we investigated their stabilizing action by determining the life of bubbles at the surface of water covered with layers of insoluble fatty acids, alcohols and esters(2). The life of a bubble under the surface of water from the moment of the formation of the bubble to the moment of the collapse of the film at the top of the bubble, may serve as a measure of the stability of a foam unit and characterizes the stabilizing effect of the adsorption layer(3).

Hardy(4), and subsequently Talmud and Suchovolskaja (5)ascertained that the maximum duration of the life of bubbles corresponds to the region of unsaturated adsorption layers. Our measurements showed that this maximum of stabilizing action actually corresponds to incomplete saturation of unimolecular adsorption films. Quantitatively, however, our data disagree with the previous investigations: e. g., according to the data of Hardy, the stabilization of bubbles by an oleic acid film begins to manifest itself and exhibits a maximum at an area $a=64$ Å^2 per molecule, whereas, according to our own data, this occurs no sooner than for $a=47$ Å^2. According to Talmud's data, palmitic acid films stabilize the bubbles and show a maximum (at room temperature) already at $a=50$ Å^2, though our measurements showed that this occurs only for $a=24.6$ Å^2 with a maximum at $a=23.6$ Å^2 on an acid substrate (0.01 n HCl); on pure water the maximum corresponds to a still smaller area per molecule. Myristic acid films on 0.01 n HCl begin to stabilize the bubbles at areas $a=42$ Å^2 per molecule and less. Such a difference between the stabilizing effects of the films of oleic, myristic and palmitic acids may be accounted for, in our opinion, by the influence of the state of the films: the first two acids give liquid expanded films, whilst palmitic acid yields liquid condensed films. As a substantiation of this opinion we found that the stabilizing action of a palmitic acid film which undergoes a transition to the liquid expanded state on heating to 28—32°, begins to manifest itself under these conditions also at 41 Å^2 and shows a maximum at $a=35$ Å^2 (Fig. 1). The essential rôle of the state of an adsorption film was further vindicated by the results obtained on adding cholesterine to oleic and myristic acids and to ethyl palmitate when forming the films. Cholesterine reduces the two-dimensional vapour pressure of the films of the above substances, and brings these films to the

condensed state. Accordingly, on raising the cholesterine concentration in the mixture, the stabilizing effect is reduced, and the stabilization isotherms $\tau = \varphi(a)$ are shifted in the direction of smaller areas. The position of all the curves $\tau = \varphi(a)$ with respect to the axis of abscissae agrees with

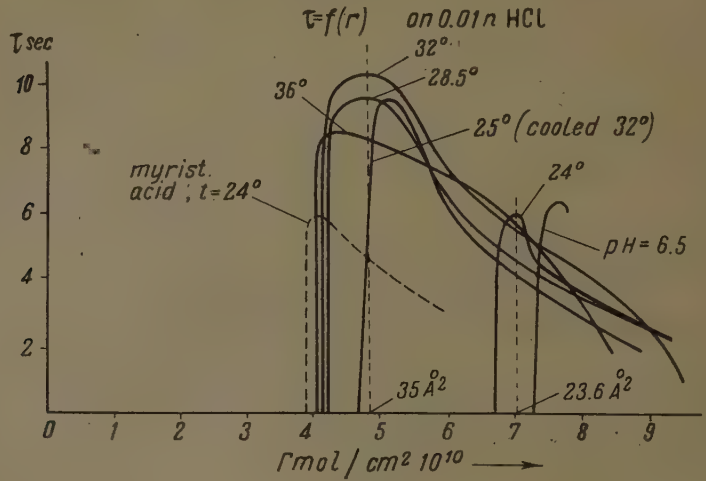

Fig. 1.

the two-dimensional pressure isotherms $F = f(a)$ obtained for the same substances by Adam(6). This seems to be an evidence for the higher reliability of our results as compared with those obtained by the previous investigators. The expansion of palmitic acid films on heating and the manifestation of the stabilizing effect at lower surface concentrations $a = \frac{1}{\Gamma N}$ are also

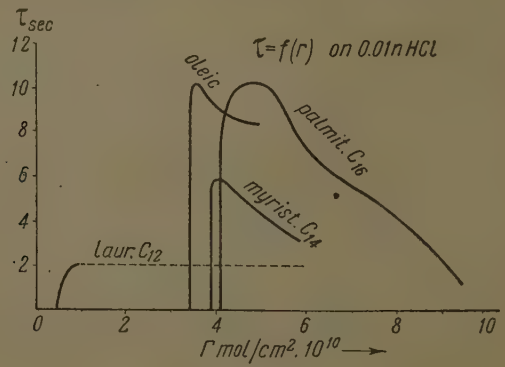

Fig. 2.

accompanied by an increase in the duration of the bubble life: $\tau_{max} = 6$ sec. for 24° and $\tau_{max} = 10$ sec. for 30°.

The stabilizing effect of the films formed by homologous compound may only be compared for films in similar states, e. g. either condensed or expanded. Such a comparison (Fig. 2), when made between palmitic acid at 30°, myristic acid at 24° and lauric acid at 24°, which at these tempe-

ratures are in a liquid expanded state (or nearly so for lauric acid), shows that the stabilizing effect sharply rises with the lengthening of the hydrocarbon chain in the molecule on transition to the higher homologous. Thus, for lauric acid, the maximum life of a bubble is $\tau_m = 2$ sec., whilst for myristic acid $\tau_m = 6$ sec. and for palmitic acid $\tau_m = 10—12$ sec. It was previously assumed that the maximum stabilization of bubbles, which lies in the region of unsaturated adsorption layers, coincides with their maximum resistance to shear, e. g. shear strength[7]. As has been shown in the first communication of this series[1] the resistance of adsorption layers to shear reaches a maximum always on complete saturation.

This shows that the stabilizing effect, i. e. the stability of foam bubbles, is not exclusively determined by the mechanical properties of the adsorption layer. This is also borne out by the values of the life τ of individual bubbles for the elastico-brittle film of palmitic acid, possessing a high tensile strength of the order of 10—12 dynes per centimetre (when spread on distilled water containing insignificant traces of polyvalent cations, e. g. of aluminium). In such a film the values of τ for individual bubbles fall under two groups one of which contains a greater variety of values, all in the neighbourhood of zero, and the other consists of a small number of values reaching 50—60 sec. Now in the case of the liquid films of palmitic acid on 0.01 *n* HCl all the values of τ are confined to the limits of from 6 to 12 sec., i. e. but little differ from each other, and so high values as 50—60 sec. are never encountered. Such a partition of the values of τ into two groups in the case of elastico-brittle films may be explained by the high brittleness of these films and their total inability to skin the gaps formed in the films as a result of the rush of water layers into the two-sided films of the bubbles, i. e. by the very low mobility of molecules in such adsorption layers. In liquid adsorption films, e. g. in those of the higher fatty acids on acid substrates, the integrity of the films is readily restored, and the life of all bubbles is approximately uniform—on an average longer than in elastico-brittle films. The occasional very high values of τ in the latter are due to the extremely robust structure of the corresponding individual portions of the films, in which the micro-fissures constitute the starting-points of collapse. Such micro-fissures in two-dimensional systems—solid films—may be treated as analogues of the micro-fissures and weak spots existing, according to Smekal, in the bulk of crystals.

The occurrence of high values of τ in elastico-brittle films as well as the increase in the stabilizing effect with the length of the chain in the molecule, accompanied by an increase in the resistance to shear in adsorption layers, points to the important rôle played by the mechanical properties of the latter in the process of stabilization. The second factor which is necessary for stabilization is the mobility of the molecules in the film, i. e. their ability to rapidly skin the gaps formed and thus restore the integrity of the adsorption layer.

The collapse of the two-sided film surrounding a foam bubble upon gradual reduction of the thickness of this film is in the nature of sharp syneresis, i. e. formation of dehydrated aggregates whose rate of reverse propagation into the film and of the attainment of thermodynamic equilibrium is slower than the velocity of destruction of the bubble. A similar interpretation is also applicable to the liquid films of, e. g., oleic acid or to the gaseous expanded films of ethyl palmitate. In spite of the absence of any appreciable hysteresis in the two-dimensional pressure on expanding an adsorption layer in a Langmuir trough, it may be assumed that hysteresis does exist in the micro-process of the collapse of a two-sided bubble film, occurring with a greater velocity. This is also borne out by the existence

of structural viscosity, i. e. hints of elasticity in the vapour films of ethyl palmitate, this factor being liable to reduce the mobility and inhibit the skinning of the gaps in the film.

To eliminate the possible error in comparing the positions of the curves $\tau = \varphi(\Gamma)$ with those representing the variation of the mechanical properties, e. g., surface viscosity expressed in terms of the logarithmic decrement of oscillation λ ($\lambda = \ln \frac{S_{n-1}}{S_n}$ where S_{n-1} and S_n are consecutive amplitude values) with increasing of surface concentration Γ, the measurements were conducted simultaneously for both curves, e. g., for the films of cetyl alcohol (Fig. 3). Although the displacement between the maxima of these curves amounts to but $a_1 - a_2 = 1.2$ Å^2 yet, even in this particular case,

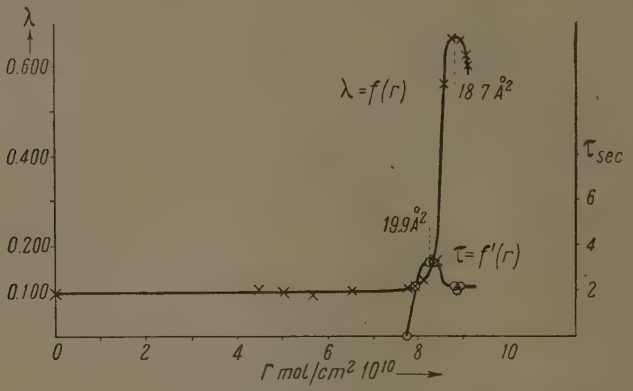

Fig. 3.

of liquid viscous films, the stability maximum occurs before complete saturation of the layer is reached.

The existence of a maximum of the stabilizing effect before the saturation of an adsorption layer may be accounted for by the fact that the stabilizing effect only appears when the film becomes continuous (in the vapour region $\tau = 0$) whilst the decrease in the stabilizing effect on approaching saturation is due to the circumstance that the aptitude for rapidly filling the clean water areas formed on collapse is lower in saturated layers in which, according to Langmuir, the attraction forces act not only between hydrocarbon chains but also between the dipoles of the polar group in the molecules of the film.

From the relation of the curves $\tau = \varphi(a)$ and $F = f(a)$ established by us it may be seen that the study of the stabilizing action of the adsorbing layers may be successfully used to judge of their condition.

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CHEMIE

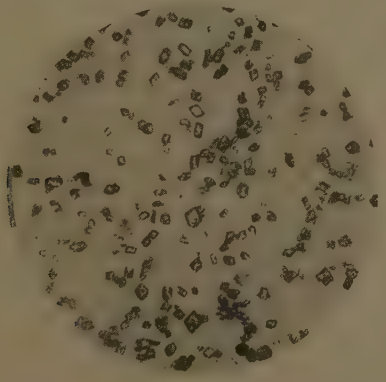
ÜBER DIE SYNTHESE VON INYOIT

Von A. V. NIKOLAEV und A. G. ČELIŠČEVA

(Vorgelegt von N. S. Kurnakov, Mitglied der Akademie, d. 7. I. 1938)

Seit 1935 werden im Institute für allgemeine und anorganische Chemie der Akademie der Wissenschaften der UdSSR Arbeiten über die Synthese von Boraten geführt. Nach der von van't Hoff(1) vorgeschlagenen und von uns teilweise abgeänderten Methodik erhielten wir folgende Borate: Pinnoit, Kolemanit, Pandermit, Boronatrocalzit und Kaliborit.

Die Synthese von Inyoit gelang nicht. Die vielen Versuche denselben aus Boronatrocalzit durch Einwirkung von Wasser oder einer schwachen



NaCl-Lösung (0.5%) bei Anätzung mit natürlichem Inyoit schlugen fehl, obwohl wir mehr als ein Jahr warteten.

Eine Erklärung für diese Tatsache brachte die Aspirantenarbeit von N. M. Selivanova im Moskauer Chemish-Technischen Institut, die unter Leitung von A. V. Nikolaev ausgeführt wurde. Es erwies sich, das Inyoit von den Calziumboraten am labilsten ist, und dessen Verhältnis zu Pandermit und Kolemanit ist derart, dass letztere bei entsprechender Anätzung sich aus ihm bilden.

Am 15 Juni 1937 begannen wir mit der Untersuchung des Systems $CaO + B_2O_3 + H_2O$ bei 25°. In einem der Reagenzgläschen traten die charakteristischen und gut ausgebildeten Kristalle des Inyoits auf. Die Bestim-

mung des Brechungsindex ergab N_p=1.491, N_g=1. 518; N_m=1. 505, was mi den Literaturangaben übereinstimmt(²).

Analyse der flüssigen Phase

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$\frac{B_2O_3}{CaO} = 13.53$	$\frac{B_2O_3}{CaO} = 14.31$
$B_2O_3 = 1.129\%$	$B_2O_3 = 1.128\%$
$CaO = 0.067\%$	$CaO = 0.063\%$

Analyse der festen Phase

	(Theorie)
$\frac{B_2O_3}{CaO} = 1.51$	$\frac{B_2O_3}{CaO} = 1.50$
$B_2O_3 = 37.85\%$	$B_2O_3 = 37.70\%$
$CaO = 20.15\%$	$CaO = 20.17\%$
$H_2O = 42.15\%$	$H_2O = 42.13\%$

Somit wurde zweifellos zum ersten Male Inyoit synthetisiert. Von Interesse ist, dass der so labile Inyoit 5 Monate lang unverändert geblieben ist; dies ist ein Beispiel für die «Labilität» des unstabilen Zustands.

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CHIMIE

SUR LA PRÉPARATION DES ÉTHERS DE L'ACIDE ORTHOSILICIQUE

Par M. N. KALININ

(Présenté par I. V. Grebenščikov, de l'Académie, le 10. I. 1938)

Il est connu que parmi les matières premières employées pour la synthèse des différents composés silico-organiques une place importante est occupée par les éthers d'acide orthosilicique qui possèdent la faculté de prendre part dans de diverses genres de réactions et de substituer leur hydroxile par des groupes variés. Les différentes méthodes pour la préparation de ces éthers sont toutes basées sur la réaction des alcools anhydres avec du chlorure de silicium, qui procède d'après l'équation:

$$SiCl_4 + 4ROH = 4HCl + Si(OR)_4,$$

R signifiant un groupe alcoyle ou aryle.

Pratiquement la préparation des éthers d'acide orthosilicique peut être accomplie à l'aide de deux procédés généraux. L'un d'eux a été proposé par Helfrich et Hausen (1) et l'autre qui est une modification du premier par Dearing et Ride (2).

Selon les indications des auteurs de la première méthode le rendement des éthers constitue 70—80% de la théorie; les auteurs du second procédé mentionnent que le rendement du tetraéthoxymonosilane constitue 70%, mais ne donnent pas les rendements des autres éthers.

En préparant de différents éthers de monosilane je me suis convaincu qu'en réalité les rendements de ces éthers sont presque toujours considérablement au dessous des rendements indiqués plus haut et s'en rapprochent seulement dans des cas rares; cette divergence dépend probablement de la grande quantité de chaleur, qui se dégage au cours de la réaction et grâce à laquelle le chlorure de silicium, ayant un bas point d'ébullition, s'évapore partiellement, sans prendre part à la réaction, même dans les cas où l'on emploie des mélanges refrigérants de sel et de glace. Le refroidissement par l'oxyde carbonique solide peut donner, d'après certaines indications, de meilleurs résultats, mais son application offre plusieurs inconvenients, surtout dans les cas où l'on est obligé de préparer les éthers de monosilane dans des quantités considérables.

Les expériences exécutées montrent que les rendements des éthers de silicium peuvent être considérablement augmentés si la réaction est menée en présence d'un dissolvant en qualité duquel nous avons employé le benzène, préalablement rectifié et séché avec du sodium métallique; en ce cas le refroidissement ordinaire à l'aide d'un mélange refrigérant est parfaitement suffisant.

La préparation des éthers de silicium a été menée par nous d'après la méthode suivante: le ballon réactionel (un matras à fond rond) était raccordé avec une allonge à trois tubulures, dans l'une de ces tubulures était introduit un agitateur, mis en mouvement par un électromoteur et pourvu d'une fermeture hermétique à mercure; la seconde tubulure était munie

d'un entonnoir à robinet et la troisième était attachée à un refrigérant dont le bout opposé était raccordé à l'aide d'un tuyau de ver, avec un tube en forme de U; ce dernier étant joint à son tour à un flacon de Tiščenko. Le ballon réactionnel et le tube étaient refroidis à l'aide d'un mélange de glace et de sel. Le flacon de Tiščenko contenait de l'acide sulfurique concentré. Le rapport entre le poids du dissolvant et celui des substances réactives était 1 : 1 et c'était d'ordinaire la solution de l'alcool dans le benzène qu'on ajoutait au chlorure de silicium dissout dans le benzène et non pas vice versa. On ajoutait la solution goutte à goutte, 75—100 g par heure. La solution étant introduite, on remuait le mélange au cours d'une heure à la température de chambre, puis on chassait le benzène et distillait le reste ordinairement dans le vide. 98% de la quantité initiale du benzène sont dégagés par ce procédé. Du tableau présenté ci-dessous on voit combien l'emploi d'un dissolvant est favorable pour le rendement des éthers de silicium.

Ethers de silicium	Rendement en pourcentage obtenu	
	moyennant le dissolvant	sans dissolvant
1. Tetraméthoxymonosilane	61	73
2. Tetraéthoxymonosilane	65	86—88
3. Tetrabutoxymonosilane	61.2	82.5
4. Tetraisobutoxymonosilane	62.4	81.6
5. Tetraisoamyloxymonosilane	54	79

Si les alcools agissant sur le chlorure de silicium ont été pris en quantité insuffisante, on obtient des chloroéthers, qui sont dûs à une substitution incomplète des atomes de chlore du chlorure de silicium par les radicaux alcooliques. En des cas pareils il se forme des mélanges difficilement séparables composés de mono-, di- et trichloroéthers et des éthers complètement substitués et, alors l'isolation du produit désiré ne peut être achevée qu'après un fractionnement fait à plusieurs reprises(3). Suivant l'indication de certains auteurs, la séparation des mélanges des chloroéthers est une opération extrêmement difficile(4). Donc, il serait intéressant d'examiner la formation des chloroéthers sous l'action d'un dissolvant. Les synthèses de triéthoxychloromonosilane prouvent qu'en présence d'un dissolvant le processus de la réaction peut être dirigé plus aisément dans la direction voulue et que dans ces cas il est possible d'obtenir le produit pur après une troisième et même après une deuxième distillation, tandis qu'un mélange de chloroéthers obtenus en absence du dissolvant ne peut pas être séparé d'une manière satisfaisante même après quatre distillations réitérées. Le rendement de $SiCl(OC_2H_5)_3$ obtenu en présence d'un dissolvant constitue en moyen 50% de la théorie.

Conclusion

1. L'emploi d'un dissolvant benzène augmente les rendements des tetraéthers de l'acide orthosilicique.

2. L'emploi d'un dissolvant (benzène) facilite la préparation des chloroéthers de l'acide orthosilicique.

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CHEMISTRY

ON THE ACTION OF SULFIDE MINERALS ON SOLUTIONS OF GOLD CHLORIDE AND CYANIDE

By O. E. ZVIAGINCEV and E. L. PISARŽEVSKAIA

(Communicated by N. S. Kurnakov, Member of the Academy, 14. I. 1938)

The action of the sulfide minerals, halenite, pyrite, chalcopyrite and sphalerite, on platinic chloride has already been investigated and published by us. (1) It was then indicated that at 20° gold salts are reduced to the metal in various ways with various velocities, viz.: pyrite and chalcopyrite reduce them quickly; halenite and sphalerite very slowly.

In this paper we present facts on the reduction of solutions of gold chloride and cyanide by these same minerals.

For the experiments $HAuCl_4$ solution, containing 0.1690 g of gold to 100 ml, was put into litre beakers, 700 ml in each. These beakers were provided with stirring rods which were all set into motion by one motor, by means of gears. All the beakers were under the same conditions (at room temperature, about 20°). One gram of sulfide, pulverized in a mortar and sifted through a 150-mesh sieve, was put into each beaker. The experiment was begun on June 9, 1936 and lasted several days. The beaker containing purite and chalcopyrite became colorless in the first few days. The first tests were made on 10 ml of each solution on June 11, and indicated the results shown in the second column of Table 1. The next tests were made on June 16 and July 2; the results of gold determinations are given in the third and fourth columns of the same Table.

The Table 1 shows that the reduction of gold by pyrite and chalcopyrite, at a temperature of 20°, from a solution of $HAuCl_4$ proceeds very quickly, while halenite and sphalerite reduce the solution very slowly*.

Table 1

	Gram gold in 100 ml solution			
	Before experiment	After 2 days	After 7 days	After 24 days
Control test (without sulfide) . .	0.1690	0.1680	0.1620	0.1600
Sphalerite solution	—	0.1380	0.1040	0.0400
Halenite »	—	0.0960	None	—
Pyrite »	—	None	—	—
Chalcopyrite »	—	None	—	—

* The composition of the sulfides has been given in our previous article.

Apparently the process of reduction with pyrite and chalcopyrite, on the one hand, and with halenite and sphalerite, on the other, does not proceed identically.

This circumstance brings up the questions of investigating the chemism of the reduction and the kinetics of these reactions. Only profound investigation of these problems will throw light upon the essence of reactions, occurring in nature.

A second series of experiments was conducted with cyanic solutions, connected with the explanation of technological problems. In 1916 an experimental investigation of the action of sulfides on gold cyanide solutions was conducted by V. A. Pasukhin [2]. The experiments were carried on at room temperature in the course of 12 hours. In this time the sulfides did not show any marked action on the solutions.

We conducted more prolonged experiments. The method was the same as with the $HAuCl_4$ solution. Table 2 shows the course of reduction of a solution of $KAu(CN)_2$ by pyrite, chalcopyrite, halenite and sphalerite.

Table 2

	Gram gold in 100 ml solution		
	Before experiment	After 20 days	After 53 days
Control test (without sulfide)	0.1830	0.1820	0.1750*
Sphalerite solution	—	0.1820	0.1730
Halenite »	—	0.1020	None
Pyrite »	—	0.0870	None
Chalcopyrite »	—	0.0840	None

* A little gold was reduced on the walls of the beaker, by the dust from the air in the laboratory.

It is evident from Table 2 that the gold cyanide solution is reduced much more slowly than the chloride solution, yet it is reduced. The chemism of the reduction is also apparently different for halenite and sphalerite, on the one hand, and for pyrite and chalcopyrite, on the other.

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CHEMISTRY

THE MUTUAL DISPLACEMENT OF METALS FROM THE VAPOURS OF THEIR SALTS AND THE APPLICATION OF THIS PROCESS IN THE FIGHT AGAINST CORROSION

By N. A. IZGARYŠEV and E. S. SARKISOV

(Communicated by W. A. Kistiakowsky, Member of the Academy, 18. XII. 1937)

Just as more electropositive metals are displaced by more electronegative ones from aqueous solutions of their salts, so metals react with the latter when they are in a gaseous state, if the system: metal A and salt of metal B is in a thermodynamically unstable state.

Thus, Martin (1) studied the reaction of the displacement of aluminium from aluminium chloride vapour with iron according to the equation:

$$2Fe + 2AlCl_3 = 2FeCl_3 + 2Al,$$

and Becker (2), Herdt and Koster studied the reaction of displacing chromium from its chloride vapours with iron.

Processes, closely related in the final result to those indicated, are the various methods of fusing certain protective metals onto the surface of those to be kept from corrosion. Of these processes of great practical value are: hot tinning, galvanizing etc.; calorizing and sherardizing, i. e. methods of coating iron in contact with the heated pulverized metal it is to be coated with. This metal is generally aluminium and sometimes zinc. The so-called «cementation» of iron in contact with pulverized metallic chromium or ferrochrome has also found some application through a limited one (3).

A phenomenon similar to the «cementation» of iron and copper with pulverized molybdenum, cobalt, tantalum, vanadium, nickel, zinc and uranium, which has not as yet found practical application, may also be mentioned. Cases are also known of copper absorbing the zinc from the vapours of the latter (4), etc.

The main task we set ourselves was to study the regularity of the least investigated process, i. e. of the displacement of metals by one another from the vapours of their salts, to fix their order in a series according to their ability of displacing, similar to that already fixed for aqueous solutions and drawn up into the table of the electrolytic intensity series.

A systematic investigation of this kind may be of importance in practice as well. It enables one to realize a whole series of protective coatings of a special type in order to find such new methods in fighting corrosion that will permit the protective metal not only to cover the surface, but also to penetrate, in the course of the replacement reaction, deeply into the metal it has to protect.

Our first experiments on displacement with chromium were carried on in the following manner.

The chloride vapours necessary in coating iron with chromium were produced in electric furnace conditions by acting on the metal with dry hydrogen chloride which is blown in with a current of hydrogen. The vapour thus obtained reacted with the iron onto which metallic chromium precipitated. Hydrogen chloride was displaced by passing pure hydrogen from the electrolyser through concentrated hydrochloric acid (sp. gr. 1. 19). The gaseous mixture produced was dried with sulphuric acid, and then led into a porcelain combustion tube, 80 cm in length and 2.5 cm in diameter, inside of the electric furnace. The specimens to be coated and electrolytic chromium consisting of little scales were placed in this tube, besides the very distribution of the materials within the tube was varied: in some of the experiments the iron specimens were placed right up on the chromium; in others the chromium was placed nearer the end the hydrogen chloride entered the tube, whereas the specimens were placed chainwise almost along the whole length nearer to the end the hydrogen chloride left the tube. After the air had been forced out with pure hydrogen and the tube had been heated for a definite length of time at a definite temperature, the mixture of hydrogen and hydrogen chloride passing through for the sake of reaction, the tube was cooled with air; after which the specimens were extracted and tested.

In the experiments the temperature of the furnace, measured with a pyrometer (thermocouple Pt—Pt-Rh), the time of heating and the concentration of HCl in the mixture were varied. The concentration of HCl in the mixture was varied by changing the temperature of the hydrochloric acid.

The velocity with which the hydrogen was passed through the tube, was kept constant (39 bubbles a minute by our counter).

The specimens we used were prepared from «Armco» iron, were cylindric in form, and had a surface area of 1. 57 cm^2 each. After having been treated with chromium, the specimens, which under the most favourable conditions acquired a light lustrous surface, were tested for their stability against a number of corrosive agents.

Besides this, the chromium coated specimens were studied under the Zeiss metallographic microscope.

While the influence of the duration of chromium displacement was being studied, the temperature was kept constant at 1100°, or around 975° for 1, 2, 3, 4 and 7 hours. The difference noted under the microscope was that the thickness of the chromium coating increased with an increase in the reaction time, and especially at a rise in temperature.

Table 1

Temperature and duration of treatment	1 hour 1100°	1 hour 975—875°	1 hour 975°
Thickness of chromium layer in mm	0.272	0.119	0.0408

Under the chromium layer an intermediate layer is formed (probably a compound of iron and chromium or their eutectic mixture), which still requires special supplementary study.

The mikrosections of the specimens are presented in the photographs (Figs 1—4).

In order to test the defensive properties of our coatings we placed the specimens in a 10% solution of sodium chloride, doing the same with unprotected specimens in other vessels. While the unprotected specimens became

completely coated with rust in the first hours of the test, corrosion of the specimens protected with chromium was limited to the appearance of individual yellow spots of rust; then, after a metallization of longer duration,

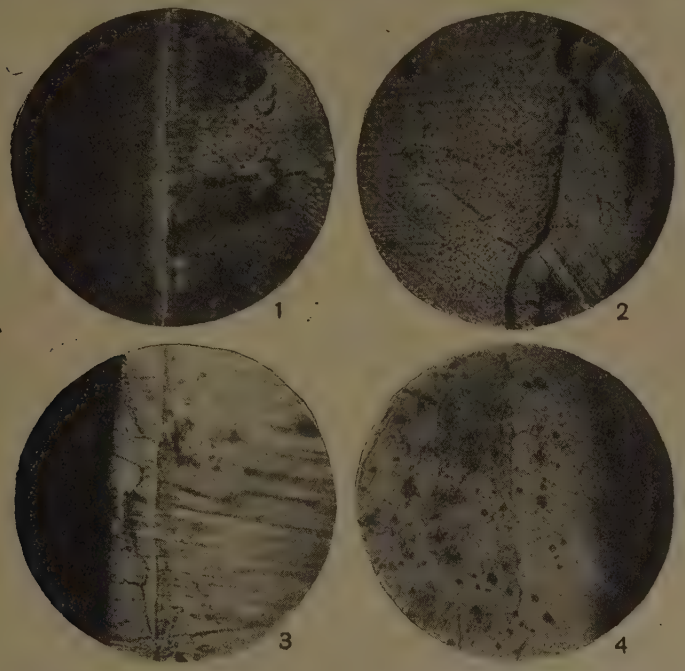

Fig. 1—Chromoplating, 1 hour, t—975°, ×500. Fig. 2—Chromoplating, 7 hours, t—1100°, ×500. Fig. 3—Chromoplating, 7 hours, t—975°—875°, ×500. Fig. 4—Chrom-coating and nickeling, 7 hours, t—975°, ×250.

such spots appeared only after 24 hours. The loss in weight of the specimens in g/cm² after being in the solution 10 days and nights was:

Table 2

Temperature and duration of treatment in chromoplating	1 hour 975°	7 hrs 975°	7 hrs 1100°	7 hrs 975—875	7 hrs 1100—875°
Loss in weight in g/cm² per hour .	$15 \cdot 10^{-6}$	$3 \cdot 10^{-6}$	$1.5 \cdot 10^{-6}$	$0.5 \cdot 10^{-6}$	$2.3 \cdot 10^{-6}$

In the same solution unprotected iron lost $19 \cdot 10^{-6}$ g. A rise in the temperature of treatment to 1100° is also good for the quality of the coating; the surface becomes more lustrous and the loss in weight in a salt solution is much less, according to Table 2.

The following manner of treatment gave very favourable results: heating up to 975° for four hours was followed by a three-hour treatment at 875°.

Even more satisfactory results were obtained in humid air in the presence of 10% carbonic acid and 10% hydrogen sulphide. In this case the specimens were hung in jars, through which the gaseous mixture mentioned was evenly drawn.

While the unprotected iron became coated with a black, liquid, off-flowing mass in the very first minutes, the specimens protected with chromium,

under the most favourable conditions, remained lustrous for 50 hours, whereupon the test was discontinued.

A similar effect was observed in humid air with an admixture of 15% sulphur dioxide. Tests in a normal solution of hydrogen chloride gave the results presented in Fig. 5, while the course of corrosion could be followed from the volume of hydrogen educed.

In Fig. 5 the velocity of dissolving the iron is represented by curve *1*; of chromium-plated iron by curve *2*; and of iron coated with chromium and nickel by curve *3*. In normal solutions of nitric and acetic acids no evolution of hydrogen was noticed. In normal sulphuric acid only 2.5 cm^3 of hydrogen was evolved in 24 hours.

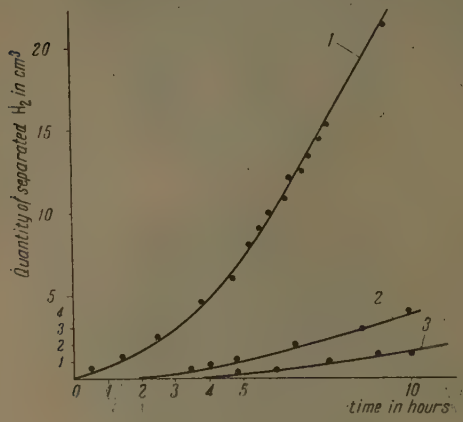

Fig. 5.

An increase in the concentration of HCl in the current of hydrogen also acts very favourably on chromium-coating. This increase is brought about by raising the temperature of the hydrochloric acid—the source of hydrogen chloride to 60°.

Our first experiments showed that the distribution of the specimens in the furnace does not guarantee their entire coating on all sides, since they touch the walls of the tube in some of their parts. The latter circumstance probably explains the appearance of individual spots, due to corrosion, on some of the specimens in the solutions.

Besides chromium displacement, preliminary experiments on the displacement of some other metals from their chlorides were made, showing the following results: tungsten gives full, lustrous coatings; and nickel dark-grey, crumbly and soft precipitated. Nickel precipitated into iron already coated with chromium gives somewhat more durable precipitates, but still not of good enough quality. Finally the precipitations of metals simultaneously from the salts of nickel and chromium, i. e. if nickel and chromium are in the tube at the same time, lustrous, durable coatings are again formed.

Their relation to corrosive agents is characterized by curve *3* in Fig. 5, which shows that iron protected with chromium and nickel simultaneously proves to be very durable against the strongest aggressor—against hydrochloric acid.

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BIOCHEMISTRY

THE NON-EQUILIBRIUM OXIDATION-REDUCTION POTENTIAL

By N. NEKRASOV

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The question as to the nature and importance of the redox-potential determined in non-equilibrium and irreversible redox-systems is nowadays one of the most actual problems of the theory of the oxidation-reduction (redox) potential. That is easy to understand if we call to mind that a large majority of biochemical and biological redox-systems are non-equilibrious. However, the theory of the redox-potentials, founded on propositions of the classical electrochemistry, can be directly applied only to reversible, equilibrious redox-systems. We have thus to state a gap between the theory of the redox-potentials and the experimental material obtained in the course of biological and physico-chemical investigations.

This gap impedes our further progress and more than once attempts have been made in literature to «theoretically conceive» the redox-potentials determined in non-equilibrium biochemical redox-systems, for example, in solutions of substances containing the sulfhydryl group (cystine-cysteine, glutathione), in sugar-solutions, etc. (R. Wurmser, L. Michaelis and others [1, 2, 3, 4])]. But these attempts have resulted in merely indefinite and ungrounded advancements.

If we want to through light on the physical nature of the redox-potential measured in non-equilibrium and irreversible redox-systems, we must call to mind the process of potential formation on an indifferent electrode immersed in an aqueous redox-system.

In aqueous media there corresponds to any potential of indifferent electrode a definite superficial concentration of hydrogen or oxygen on the electrode. However may proceed the process of saturation of the surface of the electrode, this process proceeds up to the time (or, more exactly, up to such concentrations of hydrogen or oxygen on the electrode) where it is counterbalanced by an opposite process leading the hydrogen (oxygen) off from the electrode. If the electrode reacts with a reversible redox-substance, this opposite process is also a reverse one with respect to the former. So, say, the action exerted on the electrode by the reduced form (Red) of a reversible oxidizing-reducing agent (for instance, hydrochinon), leads to an increase of hydrogen concentration on the electrode, while the action of the corresponding oxidized form—Ox—(for instance chinon) results in a decrease of that concentration. In the course of their «reactions» with the electrode the Ox- and Red-forms undergo a mutual transformation which phenomenon is to be regarded as a fundamental characteristic of the true (labile) chemical equilibrium. The process is quite different in the case of irreversible systems: the Ox-form of cysteine—cystine for example is not reduced under ordinary conditions and, although the Red-form of cystein displaces the electrode-potential in the negative direction and performs consequently the same function as the Red-form of any reversible redox-substance, the processes limiting this potential—displacement can obviously not result

in the regeneration of the primary Red-form, being therefore not inverse, but only opposite with respect to their action on the electrode. Thus on the latter a typical stationary condition is created.

Let us now examine one of the most simple examples of a redox-system, characterized by a complete absence of reactionable Ox-forms. The electrode potential can be displaced in this case in the positive direction: first, under the influence of molecular oxygen (if the latter is present in the electrode surrounding medium, or dissolved in the metal which it is made of), and secondly, owing to a decrease in the amount of hydrogen on the electrode which may be due to direct diffusion of hydrogen into the surrounding medium, or recombination into H_2-molecules, followed by their subsequent diffusion.

The electrode potential attains a stable value when the summed up velocity of the above processes becomes equal to the velocity of the potential displacement in the negative direction by the non-equilibrious Red-form, e. g. by cysteine.

In accordance with the above let us assume that

$$E=\frac{RT}{F}\ln\frac{[H+]}{aC_H}+E_0, \qquad (1)$$

where E—the electrode potential; a—the coefficient of proportionality; C_H—the hydrogen concentration on the electrode, and E —the normal potential*.

The dependence of E on the different factors can be expressed as

$$\frac{dC_H}{dt}=k_1[\text{Red}]-k_2\cdot C_H[O_2], \qquad (2)$$

k_1—the velocity constant of the reaction $\text{Red}\rightarrow(\text{Ox})+H$, where (Ox) represents the inactive form; k_2—the velocity constant of the oxidation reaction between hydrogen and oxygen on the surface of the electrode. After a stationary condition, i. e. a limiting potential, has been established, the following relations are valid: $\frac{dC_H}{dt}=0$, i. e. $k_1[\text{Red}]=k_2\cdot C_H[O_2]$; $C_H=\frac{k_1[\text{Red}]}{k_2[O_2]}$, and consequently

$$E=\frac{RT}{F}\ln\frac{k_2(O_2)}{k_1[\text{Red}]}+\frac{RT}{F}\ln[H^+]+E_0. \qquad (3)$$

It may appear at first sight that there exists no difference between the above equation and the usual thermodynamic equation of the same process, provided that the part of the Ox-form is played by oxygen. In reality there does exist an essential difference.

It is known that precisely in the case of a reversible reaction the relations between the velocity constants of the direct and inverse process can be expressed by the equation $\frac{k'}{k''}=K$, where K is the constant of the reaction equilibrium $K=\frac{[\text{Ox}][H]}{[\text{Red}]}$. This relation ensues from the mass law.

Hence the equation for E can be transcribed as follows:

$$E=\frac{RT}{F}\ln\frac{k''[\text{Ox}]}{k'[\text{Red}]}+\frac{RT}{F}\ln[H^+]+E_0=\frac{RT}{F}\ln\frac{[\text{Ox}]}{K[\text{Red}]}+$$
$$+\frac{RT}{F}\ln[H^+]+E_0=\frac{RT}{F}\ln\frac{[H+]}{[H]}+E_0. \qquad (4)$$

---

* The stoichiometric ratios of the reaction between the hydrogen adsorbed on the electrode and the oxygen do not matter to the object of our reasoning. The second term of the right part of the equation is, therefore, given the simplest possible expression.

Thus it is obvious that the final potential of an equilibrium redox-system is not influenced by kinetic factors.

As to the equation given for the non-equilibrium system, there exists, as a rule, no connection between the constants k_1 and k_2, and therefore they cannot be elimitated from the equation deduced for E.

The situation we have before us when determining E in a non-equilibrium redox-system after the indicator method may be analysed by expressions of the same type as used by O. Warburg and Christian(5) for the deduction of the relation existing between the degree of oxidation of the «yellow oxidizing ferment» and the oxygen content of the liquid.

Let be: C—the total concentration of the redox-indicator, equal to $C_{Ox} + C_{Red}$, [Red]—the concentration of the reduced form of the irreversibly oxidixed substance, and $[O_2]$—the concentration of oxygen. We get for the stationary state:

$$\frac{C_{Ox}}{C} = \frac{k_2 [O_2]}{k_1 [Red] + k_2 [O_2]}, \qquad (5)$$

where $\frac{C_{Ox}}{C}$ is the degree of colouring of the indicator (Färbungsgrad) after which, according to the indicator method, the redox-potential is calculated. From the above equation it may be seen that in non-equilibrium systems this quantity depends directly on the velocity constants of the reactions taking place between the indicator and the redox-components of the system.

Thus, in whatever way we measure the quantity of the redox-potential in a non-equilibrium system, the value obtained will necessarily reflect not only the concentrations and relationships of the redox-components, but also the kinetic characteristics of the system which are identical with the velocity constants of the corresponding redox-reactions. From the dependence of the velocity constants on the concentrations of the «accelerators»—catalyzers and ferments follows that an addition to the system of a ferment, accelerating the splitting of hydrogen off the reducing agent, will displace the potential in the negative direction exactly in the same way as an increase of the Red-concentration*.

The question suggests itself: may this dependence be observed in the experiment? Fortunately, one is able to give a positive answer. Although the lack of definite theoretical interpretation of the nature of non-equilibrium redox-potentials prevented a systematic experimental study of non-equilibrium redox-systems, we can point out a number of observations made with other purpose in view which show an obvious dependence of the redox-potential measured in stationary (and consequently non-equilibrium) systems on the velocity of corresponding redox processes. For example, it has been established that:

1) The addition to cysteine solutions of substances known as catalyzing their oxidation (dehydrogenation) results in a decrease of the limiting redox-potential (1, 3).

2) The addition to acetaldehyde solutions of dehydrase preparations has the same effect (1, 3).

3) The same dependence is observed in a number of experiments on intercellular redox-potentials (4, 6, 7). The experimenters added to aqueous cell suspensions very small quantities of substances endowed with a desactivating power with respect to one or the other of the different redox-enzymic cell systems. It has been found that the reagents inhibiting the oxidizing enzymes (KCN, CO, H_2S) greatly depress the intracellular redox potential in aerobic conditions but do not exert any influence in anaerobic conditions.

* It should be noted that strictly speaking the composition of a stationary redox-system can remain unchangeable only under the condition that the «sources» of oxidation and reduction are not subject to a decrease. In many cases oxygen is supplied by the atmosphere which may of course be considered as a permanent source. The source of the reducing agent is limited and can be considered as permanent only within a relatively short period of time. However, in biological systems, as, for instance, in bacteria cultures, the vital activity of the microorganisms and tissues may produce new and new quantities of substances endowed with reducing ability.

The reagents inhibiting the dehydrogenases (ethyluretane, sodiumpyrophosphate) displace the aerobic limiting potential in the positive direction. Analogous results were obtained by Korr[8] who determined the redox-potential in a culture of phosphorescent bacteria, and by other experimenters.

The equation for E, which has been discussed above under the aspect of formation of a definite potential on the electrode, is valid also in the absence of any electrode. In this case C_H will express the concentration, or (in the case of sufficiently strong positive potential values) the «probability of appearance» of hydrogen in the solution itself. The stationary (limiting) redox-potential represents the measure of the a c t u a l work which can be performed by a system in the course of a redox-reaction, provided that the values of the velocity constants remain the same as during the potential determination. It is evident that this work cannot be equal to the change in the free energy of the system but it expresses nevertheless the a c t u a l intensity of the oxidizing and reducing action which can be developed by the system at the given redox-potential concentrations and k i n e t i c c h a r a c t e r i s t i c s.

The above conclusions comprise not only redox-systems. They are also valid for reversible redox-systems containing s l o w l y r e a c t i n g components, as, for example, for the redox-couples known in the biological physico-chemistry (succinic acid—fumaric acid; tartaric acid, lactic acid, and others) if those systems are activated by the corresponding enzymes. With the aid of equations analogous to the aforecited, it can be easily shown that the redox-potential of such systems must be stationary and will therefore markedly depend upon the kinetic factors and concentrations of O_2, provided $k'_2\,[Ox] \leqslant k_2\,[O_2]$, where $[Ox]$ is the concentration of the oxidized component, e. g. fumaric acid, acetaldehyde, and others, and k'_2, the velocity constant of the reaction $Ox + nH \rightarrow Red$.

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GEOLOGY

THE LOWER SILURIAN GEOSYNCLINAL PHOSPHORITES IN THE NORTH-EASTERN BRANCH OF THE KARATAU RANGE

By P. L. BEZRUKOV

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In the spring of 1937, the present author was requested by the Scientific Institute of Fertilizers and Insectofungicides to verify the information regarding the discovery of high-grade phosphorites, different from the usual nodular phosphorites, in the north-eastern branch of the Karatau range (south Kazakhstan) at the base of the Lower Silurian. The phosphorites had been found in the course of a study of the rocks collected by the geologist I. I. Mashkara in the summer of 1935, in the Kyr-Chabakty river basin, Mashkara had given a detailed description of the samples of phosphorites in his collection and had established their stratigraphic position(3), but he had hardly been able to give any information with regard to their structure, thickness, distribution and, especially, the practical value of the phosphorite-bearing horizon.

The search work carried out by the present author showed that thick composite phosphorite-bearing stratum is widely developed in the north-eastern portion of the Karatau range, the largest, economically important accumulations of phosphates being concentrated not along the Kyr-Chabatky river, but considerably further to the north-east.

The rocks of the Lower Paleozoic, crumpled in sharp folds with a north-western strike, play the chief part in the structure of the north-eastern branch of the Karatau. The sandstones and silicious schists of the Karoy series are exposed in anticlines, this series being (according to I. I. Mashkara) referred to the Middle Cambrian; the synsclines are composed of thick strata (up to 3 000 m) of limestones and dolomites of the Tamdy series, in the upper part of which V. N. Veber has collected fauna of the middle horizons of the Lower Silurian. The Lower Silurian age is arbitrarily ascribed to the lower portion of these strata. As a rule a normal stratigraphic contact of the Silurian and the Cambrian may be observed only along the north-eastern sides of the anticlines, since the south-western side are usually complicated by sharp overthrusts. A study of the contact shows that the Silurian sediments lie transgressively on the Cambrian, with traces of erosion and, occasionally, with an angular unconformity; in the south-western direction they pass from young to older rocks.

The phosphorite-bearing bed lies at the base of the Silurian, not at the boundary with the Cambrian, as it had been supposed by Mashkara, but separated from the latter by a thin (3—12 m) pack of dolomites, containing

in their lowermost position a conglomerate of pebbles of Cambrian schists. The rocks of the phosphorite-bearing bed overlie the dolomites also with an erosion, but pass onto Cambrian schists very seldom. Phosphate strata dip at angles of 25 to 90°.

The most extensive outcrop of phosphorite strata (over 100 km) is found along the north-eastern side of the Malo-Karoy and the Koksui anticlines, the latter being a continuation of the former. In the marginal south-eastern part of this band along the Tamdy and Koktal rivers, the srtucture of the phosphate series is the simplest being composed usually of one phosphorite seam, 0.2—0.8 m thick, concentrated by terrigenous material. Further north-west in the basin of the rivers Charbakty and Berkuty, its structure becomes more complicated, the thickness increases up to 10—30 m, siliceous rocks and limestones being considerably developed in the beds. This is attended by an increase in thickness and quality of the phosphorites themselves. Finally in the extreme north-western part of the region along the rivers Kok-su and Uch-bas, the total thickness of the bed reaches 50—70 m, and the total thickness of the high-grade phosphoride strata is up to 20—30 m. (The quantity of strata varies from 2 to 7; the thickness, from 1 to 14 m.) Further north-west, the rocks of the Lower Palcozoic disappear under the Cenozoic deposits, but it is possible that under their cover the phosphorite strata stretch further over a considerable area and that the above-mentioned thicknesses are not ultimate.

South-west of the belt examined, i. e. across the strike of the folds the thickness of the beds decreases. In the south-eastern part of the region the phosphate rocks passing to the Bolshe-Karoy anticline entirely pinch out, which is probably due to their approaching the shore-line of the basin by which they had been deposited. In the north-west, however, when passing to the Uch-bas anticline (for a similar distance) the rocks retain a considerable thickness (up to 12 m). Thus the isolines of facies and thicknesses intersect the strikes of the folds at a certain angle here. Still further south-west, in the principal branch of the Karatau range the Lower Silurian is represented by terrigenous formations. As yet we may hardly assume the presence of phosphate rocks among these formations. North-east of the Malo-Karoy anticline the thickness of the phosphorite beds first somewhat increases, and then in the next anticline it is reduced to 10—5 m. The above data show that the belt of intensive phosphate accumulation in the central part of this branch of the Karatau coincided in general with the Aktau syncline, pinched out as it seems, in the south-east, and widened in the north-west, enclosing the adjoining folds. If we smooth out the complicated fold overthrust structure of the range, we shall see that the width of the phosphate accumulation zone attained here at least 40—50 km.

Passing to the composition of the phosphate beds we shall first examine the phosphorites. The latter present compact rocks of a gray or black color, and at the surface of a bluish color, occurring in regular beds, and occasionally slightly laminated. The structure of the phosphorites is micro-grained oolitic, less frequently, pisolitic or homogeneous. The substance both of the grains and the cement chiefly consists of phosphate; small quantities of secondary chalcedony, calcite and dolomite, sparce grains of quartz, as well as organic substance, pyrite and iron oxides are present in the rocks. Glauconite grains are absent in typical phosphorites and are encountered only in arenaceous varieties in places where the beds pinch out. Conglomerates of the same phosphorite pebbles with a calcareous or phosphate cement are sometimes encountered along with normally stratified phosphorites. It is impossible in the present paper to discuss in great detail the structure and mineral composition of the phosphorites. Analyses of the phosphorites

show their high quality. The P_2O_5 content in the principal beds varies from 25 to 36%, and is 28—32% on the average; the average R_2O_3 content is 2%.

The phosphorites are interbedded and locally replaced along the strike by slightly phosphorized limestones, silicites and clayey shales, containing 1—2 to 18—20 per cent P_2O_5. Ironstones and ferruginous dolomites of a pea-shaped pisolitic or thin-bedded structure, 0—2.5 m in thickness, occur in the basin of the Chabakty and Berkuty rivers over the chief phosphorite layer. It should be noted that these rocks as well as the phosphorite conglomerates and the pisolitic phosphorites are developed most frequently along the margins of the phosphate bed area; evidently their deposition took place under more shallow-water conditions.

Apart from the shells of Radiolaria and the spicules of sponges discovered in the microsections of arenaceous varieties of phosphorites, no organic remains have been found in the rocks of the phosphorite beds. This fact as well as certain lithological peculiarities of the beds seem to be an evidence that the physico-chemical conditions, which predominated in the Silurian basin at the time of phosphate rock accumulation, were unfavourable for the development of bottom organisms.

The study of the composition, the mode of occurrence and the distribution of the Lower Silurian phosphorites of the Karatau range makes it possible to regard them as normal marine sediments of the Caledonian geosyncline. So far no true bedded phosphorites of the geosynclinal type similar to those composing the largest deposits in the world (1) was known in the USSR. Even the Lower Permian bedded phosphorites discovered recently in the Urals refer on account of their insignificant thickness to the common platform type phosphorites (2). Only with the discovery of the thick phosphorite beds in the Karatau we may state the presence of typical geosynclinal phosphorites in our country. A comparison of the Karatau phosphorites with the geosynclinal phosphorites of North Africa and the Western States of the USA(4,5) shows that they are almost equivalent to the latter as to quality and even superior to them as to thickness.

Deposits	The Karatau		North Africa		Western States of the USA Idaho
	Kok-su	Uch-bas	Tunis Gafsa	Marocco	
Age of the phosphorite-bearing bed	Lower Silurian	Lower Silurian	Paleocene	Maestricht-Paleocene	Lower Permian
Thickness of the phosphorite-bearing bed (meters)	50—70	52	80—100	40—60	23—56
Total thickness of the principal phosphorite layers (meters) . . .	12—20	Up to 29.5	Up to 10	Up to 10	Up to 9
Number of the principal phosphorite layers (meters)	2	7	2—10	4—8	1—4
Average P_2O_5 - content (per cent) . .	28—32	26	27—29	30—36	30—35
Average R_2O_3 - content (per cent) . .	2	2.7	2	About 2	About 2

Thus the Karatau deposit belongs in the largest phosphorite deposits of the world and is to be regarded, on the same level with the Khibina apatite deposit, as a new large phosphate stock of the Union.

It should be noted that with regard to the nature of the enclosing rocks, the outward appearance, the microstructure and, seemingly, the mineralogical composition, the Karatau phosphorites rather closely approach to the Lower Permian phosphorites of the Rocky Mountains. Though they

are much more ancient and, as far as we know, present the oldest great accumulation of sedimentary phosphates on the earth's surface. The study of the conditions under which the Karatau phosphorites were formed may be begun now on the basis of extensive data on the lithology of the phosphorite-bearing beds. From such study we shall be able, as it seems, to draw some conclusions on the genesis of the geosynclinal phosphorites of other countries, which is at present the subject of most contradictory opinions.

In conclusions it should be emphasized that the discovery of large phosphorite deposits in the Lower Silurian of the Karatau range is an evidence for the probability of finding bedded phosphorites in a number of other geosynclinal regions of the Union. At the same time it is a testimony against the suggestion of some geologists which believe that the epochs of intensive phosphate accumulation refer to later geological periods.

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MINERALOGY

ON THE TETRAHEDRAL STRUCTURE OF MINERALS

By A. E. FERSMAN, Member of the Academy

1. Almost fifty years have elapsed since (1891) V. I. Vernadsky (1), Member of the Academy, advanced the idea of alumosilicate complexes and at the same time traced a way for an analogous determination of the structure of alumosulphates, alumophosphates etc. This idea of rare intuition has nowadays been confirmed by the X-ray analysis of silicates and phosphates. The present report deploys and supplements the statements of V. I. Vernadsky, made by the latter in his report to the Academy of Sciences in December 1937.

2. Having examined the chemical composition and the structures of 1500 mineral species(2) I prepared a list of all the ions (about 150) known in these, and particularly of complex anions. I was able to perform this taking for basis the crystallo-chemical analysis of the lattices and «islands» [according to K. Weissenberg's (3) interpretation] included in these, as well as the chemical and physico-chemical data available.

Among the complex ions an abundance of tetrahedral structure is striking. They are given in Table 1.

Table 1

Tetrahedral Complex Anions

w—of the complex anion	Complexes	w—of the centr. ion
$w = 1$	$[ClO_4]$ $[JO_4]$ $[ReO_4]$	VII
$w = 2$	$[SO_4]$ $[SeO_4]$ $[TeO_4]$ $[CrO_4]$ $[MoO_4]$ $[WO_4]$ $[UO_4]$ $[ReO_4]$.	VI
$w = 3$	$[PO_4]$ $[VO_4]$ $[AsO_4]$ $[SbO_4]$; probably $[NbO_4]$ $[TaO_4]$ . . .	V
$w = 4$	$[SiO_4]$ $[TiO_4]$* $[MnO_4]$* $[ZrO_4]$* $[HfO_4]$* $[ThO_4]$* $[TRO_4]$** $[GeO_4]$ $[SnO_4]$	IV
$w = 5$	$[AlO_4]$ $[TRO_4]$ $[FeO_4]$ $[TiO_4]$ $[MnO_4]$ $[CrO_4]$ $[VO_4]$ $[GaO_4]$; probable in silicates $[BO_4]$	III
$w = 6$	$[BeO_4]$, $[FeO_4]$, $[ZrO_4]$, $[MgO_4]$	II
$w = 7$	could be produced only by Li?	I

w—valency of the ion.

* For Ti and its analogues the coordination can be 4 only at high temperatures (Kunitz Barth). Otherwise, it is 8 or 6. For B and Al the coordination is 4 or 6. For Nb and Ta the coordination is 6 or 4 only if these ions are combined with other complexes.

In complex ions a replacement of O by F^{-1} and OH^{-1} is probable. We have to point out the tetrahedral structure of neutral molecules too, playing a considerable part in geochemistry, as $SnCl_4$, TiF_4 etc.; even $(AlCl_3)_2$ is combined of two tetrahedra with a common edge.

** TR means the ions of the rare earths group (15 elements).

In Table 1 its regularity and a strict observance of the rules of valency both of the central cation and the complex itself is particularly striking. Only the complex septivalent anions, with a too low energy and, consequently, with too large central cations, and the anions of niobium, tantalum and, partially, boron arise some doubts. A total of about 50 complex oxygen anions built after the tetrahedral type may be numbered, which makes over 75% of the anions known in nature ([4,5,6,7]).

The coordination number of these structures is 4 but for some of the above mentioned atoms, the possibility of other coordination numbers is not excluded.

Alongside of the tetrahedral anions the very stable anions of the octahedral type, 6-coordination and plat triangles: $[NO_3]^{-1}$, $[CO_3]^{-2}$ and $[BO_3]^{-3}$ play a considerable rôle in geochemistry. The pyramidal complexes, e. g. $[NH_4]^{+1}$, are far less important.

3. Let us consider the manners in which the above mentioned anions combine:

1) A combination of homogeneous tetrahedra in chains, ribbons, sheets, framework with a gradual decrease of the number of free valencies is most typical of silicates.

2) A combination of heterogeneous tetrahedra, the basal tetrahedra being as a rule replaced by tetrahedra of other ions, for the most part, with a gain in energy.

The following combinations are the most common*:

1) $[SiO_4]$—combines with tetrahedra (at high temperatures) with Ti, Zr, Th, TR, Ge, Sn, tetravalent, B, Al, TR, Fe, Ti, Mn, Cr, V, trivalent, which is favourable in respect of energy and leads to the preservation of one free valency in the framework; combinations of tetrahedra with P, Sb, S, As or Nb and Ta are very rare.

2) $[PO_4]$ and its analogues usually combine with the tetrahedra with Al, Fe···, U.

3) $[NbO_4]$ and $[TaO_4]$ are caracterized by combinations with $[TiO_4]$ and $[TRO_4]$.

4) $[SO_4]$, by combinations with $[AlO_4]$ and $[FeO_4]$.

5) The complex combinations of the ions $[CrO_4]$ with $[IO_4]$ or $[MoO_4]$ with $[WO_4]$ etc. formed a separate group.

4. Table 2 shows that the central ions of the tetrahedra may have a very different valency, the maximum stability in tetrahedra being observed in ions of the size 0.3—0.4 Å. Ions of smaller dimensions make up flat triangles with a three-coordination. Ions of larger dimenions from 0.4 to 0.7 Å form tetrahedra with four-coordination only under certain conditions (at elevated temperatures); the usual coloring of these complexes in a number of combinations (e. g., Ti, Sn) shows that they are considerably less stable.

The tetrahedral ion may be arbitrarily identified with globular ions, assuming that the radius of the sphere is equivalent to the diameter of oxygen ion + the radius of the central ions, the polarization being taken into account. On the basis of an analysis of the distances actually observed between the oxygen ion and the central cation in tetrahedra, the value R of the complex anion may be obtained, varying generally around 3 Å with a deviation ± 0.5 Å. This explains, from the geometric viewpoint, the mutual replacement of these tetrahedra in the lattices. The *VEK*'s values show, however, that these ions are far from being equivalent with regard to energy, which is the reason of their greater or smaller ability of mutual replacement.

The weakest, from the standpoint of energy, and, consequently, the most acid are the ions $[IO_4]^{-1}$ and $[ClO_4]^{-1}$; on the contrary, the strongest, but either with a very slightly pronounced acidity or with amphoteric pro-

* I consider only the combinations of tetrahedral anions and omit their combinations with flations $[BO_3]$ and $]CO_3]$, which are also of considerable importance in geochemistry.

Table 2

Table of the Principal Complex Anions of the Tetrahedra Type

	Complex anions			Central cations		Color	Notes
	$-w$	R	VEK	$+w$	R		
IO_4 . .	1	3.55	0.14	7	?	Light yellow	
ClO_4 . .	1	2.36	0.21	7	?	Colorless	
SO_4 . .	2	2.95	0.35	6	0.34	»	
CrO_4 . .	2	3.00	0.35	6	0.3—0.4	Bright yellow	Strong chromophore
MoO_4 . .	2	3.45	0.30	6	?	Light greenish-yellow	
WO_4 . .	2	3.52	0.30	6	0.68	Fallowish yellow	
UO_4 . .	2	—	—	6	?	Yellow-green	Strong chromophore
PO_4 . .	3	3.00	0.50	5	0.35	Colorless	Weak polarization
AsO_4 . .	3	2.95	0.50	5	0.3—0.4	»	
VO_4 . .	3	?	?	5	0.40	Yellow-orange	
SiO_4 . .	4	2.90	0.70	4	0.39	Colorless	The Si—O distance varies from 1.58 to 1.64
AlO_4 . .	5	3.15	0.80	3	0.57	»	In silicates and phosphates
FeO_4 . .	5	3.25	0.80	3	0.67	Apple-green	
BO_4 . .	5	?	?	3	0.20	Colorless	Weak polarization of admixtures
BeO_4 . .	6	2.96	1.00	2	0.34	»	Weak polarization of admixtures
$[MgO_4]$.	6	3.40	0.90	2	0.78	»	

Notes: The *VEK*'s are given with an approximation to the second decimal place; *R* of cations with the usual 6-coordination; to express it in terms of the 4-coordination, an average 6 per cent diminishing is required.

perties, are the complexes (BeO_4) etc. The increase of the pH of natural geochemical processes facilitates the formation of tetrahedral complex anions.

5. Apart from the oxygen anions a number of sulphoanions of the tetrahedral type are known, in which a rôle is played by the more strongly polarized ion S^{-2}. Ions of still stronger polarization, Se and Te, prevent the formation of complex anions and make up simple selenides and tellurides.

The central cations available are As^{+3}, Sb^{+3}, Bi^{+3}, whose radii are unknown.

The following complex sulphoanions are less common among the tetrahedral ions: $[AsS_4]^{-3}$ and $[SbS_4]^{-3}$ with As^{+5} and Sb^{+5} at the centre; more important are: $[SnS_4]$, $[GeS_4]^{-4}$, $[FeS_4]^{-4}$.

From the analysis of physical properties the radii of these complexes should be about 2.8^{-3} Å, and their *VEK's*, of the order of 0.6—0.7, which is an evidence of their slight acidity and low hardness (2—3 according to Moos scale).

6. From the chromatic viewpoint the tetrahedral anions (8)should be regarded as very stable, and indeed they produce colorless or slightly combinations with globular cations.

This stability of the tetrahedral structure with regard to the external

field is clearly seen from a comparison of the color of simple cations with that of corresponding complex anions (Fe^{+3} red-brown and $[Fe\cdots O_4]$ green-yellow). In geochemistry the tetrahedra with central ions of the elements of the intermediate type (with free internal shells) ([9],[10]) are strongly colored chromophores.

A combination of tetrahedra of sulphoanions produces semi-metallic lattices, when a strong polarization of sulphur takes place.

7. The sequence shown in Table 2 is not only of a geometric and energetic significance, but also corresponds to the geochemical sequence of crystallization in the earth's crust namely (in ascending order):

The gradual decrease of the valency of the complex anion. The increase of its radius and the corresponding decrease of the radius of the cation of the internal sphere.

The decrease of energy of the ions in the lattice, their disappearance according to the descending degrees of energy. The increase of the odd valencies and the increase of acidity (or basidity).

The process of magmatic crystallization begins with the isolation of geometric islands with magnesium and iron. The silicious tetrahedra in a pure state are followed by their combinations with pentavalent tetrahedra, particularly with aluminium, manganese and iron, which results in the formation of a stable linkage with monovalent and bivalent globular cations.

In the course of the pegmatite process the silicates are followed by trivalent phosphates; their crystallization, with that of sulphate tetrahedra, extends to the region of hypergene processes.

To the K geophase we refer the beginning of the crystallization of bivalent tetrahedra (except the much earlier crystallization of $[WO_4]$), and finally under the final conditions of hypergene processes (in deserts) lower valencies of complexes are predominant, especially the odd ones: 1, 3, 2; under extreme conditions of a high hydrogen potential and of extreme pH values, combinations with the coordination number 3 (CO_3, BO_3, NO_3 etc.) are added to compounds with the coordination number 4.

8. From the abovesaid it follows that the earth's crust is chiefly composed of oxygen complex anion tetrahedra and corresponding sulphoanions. This is not only an oxygen packing of the most close type ([11]), but also a packing subordinate chiefly to the tetrahedral four-coordination.

The chief part from the standpoint of energy is played not by the voluminous oxygen ions, but by the multivalent small cations with strong electric fields, lying at the centre of the tetrahedra. About 85% by weight of the earth's crust belong to lattices of this type.

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HYDROBIOLOGY

ORGANIC MATTER IN SEA WATER AND FOAM IN THE SOUTH-EASTERN PART OF THE CASPIAN SEA (OCTOBER-NOVEMBER 1936)

By N. A. SKOPINCEV

(Communicated by S. A. Zernov, Member of the Academy, 29. I. 1938)

The paper presents a continuation of the author's investigations carried out in the Barentz and Kara Seas in 1932—1933 ([1,2]). In the present investigation use has been made chiefly of the method of determination of the «oxidability» in a neutral as well as an alkaline medium, with a iodometric end([3]). In addition, in some points a determination was also made of the amount of oxygen consumed in the biochemical processes involved in the incubation of water during a definite period of time (biological consumption of oxygen). Foam was also subjected to study. In the papers published on the Caspian Sea, data on the oxidability of water are given only by A. Lebedincev ([9]) who found that the oxidability of the latter ranges from 2.9 to 8.6 mg of O_2.

I. Sea Water. A. Oxidability

The work was conducted in three separate regions: a) the Krasnovodsk Bay, the coastal region; b) the open sea, and c) the Karabugaz Bay.

In considering the data obtained (Table 1) for the Krasnovodsk Bay and the Caspian Sea, we see that

1) the oxidability of the surface layers of water increases in the coastal zone, which is quite natural; it is in this zone that a considerable development of attached organisms takes place; moreover, here the contamination from the coastal villagea is felt, as well as (to a small extent) the mainland drainage.

2) The distribution of the values of oxidability in the open sea according to separate levels indicates a decrease of the oxidability with depth, the greatest decrease taking place between 25 and 50 m, i. e. at the lower boundary of the photosynthetic layer (at 25 m pH=8.53, and at 50 m pH=8.12 and 8.20). Such a course of oxidability is in good agreement with the distribution of the planktonic populatlon throughout the depth of the sea water.

3) The samples of the bottom water (Stations № 39—42) have a high value of oxidability. This is due to the accumulation of organic matter on the bottom, common for shallow depths.

4) In order to separate the organic matter dissolved from the one suspended, filtration was used through gas № 25 and through a membrane filter([4]), and the water was treated with a $Al_2(SO_4)_3$ solution [a precipitate of $Al(OH)_3$

Table 1

Results of Determination of Oxidability of Sea Water in mg O_2 per 1 litre*

	№ of station and date	Depth of station, meters	Depth at which the sample was taken, meters	°C	S‰	Oxidability			Supplement
						Alkaline medium	Neutral medium		
							Absolute	% of the alkaline	
				a) Krasnovodsk Bay					
18	25.X	0.7	0.35	15.5	14.94	7.06	3.88	53	Saimonov creek 10 m from the shore
24	11.XI	1,5	0.3	11.7	14.49	—	2.21	—	Muraviev's creek
37	22.X	1.5	1.0	12.2	14.40	3.57	2.18	61	from on brige board in the town
39	23.XI	6.5	0	12.5	—	3.33	1.86	56	5 miles south of Ufra
39	23.XI	6.5	6.0	12.5	14.06	3,36	1.98	59	
40	26.XI	6.5	0.5	12.7	13.82	—	1.79	—	Near the end of the Krasnovodsk spit
40	26.XI	6.5	6,0	12.4	13.95	—	1.91	—	
				b) Open sea					
42	26.XI	25	1	16.2	13.04	3.00	1.64	55	12 miles west of the southern part of the Krasnovodsk spit
42	26.XI	25	24	14.7	13.78	3.16	1.79	57	
42	26.XI	25	1	—	—	—	1.36	—	The same water but treated with $Al_2(SO_4)_3$
42	26.XI	25	24	—	—	—	1.36	—	
43	2.XII	450	0	17.5	13.02	—	1.72	—	75 miles west of the southern spit of island Cheleken
43	2.XII	450	25	13.72	12.92	2.77	1.60	57	
43	2.XII	450	50	7.32	12.78	—	1.25	—	
43	2.XII	450	100	6.48	12.78	—	1.13	—	
43	2.XII	450	300	—	12.70	—	1.19	—	
43	2.XII	450	420	5.92	11.30	1.91	1.05	55	
44	3.XII	130	0	17.5	12.98	2.81	1.60	57	
44	3.XII	130	25	13.71	12.91	2.57	1.48	57	62 miles west of the southern part of island Ogurchinsky
44	3.XII	130	50	8.81	12.75	2.38	1.33	56	
44	3.XII	130	125	6.42	12.80	2.18	1.29	59	
				c) Karabugaz Bay**					
27	7.XI	6.5	0	12.7	126.27	25.4	13.8	54	15 miles north of the village of Kizyl-Kup
28	8.XI	6.5	0	12.2	126.22	25.4	14.2	56	
28	8.XI	6.5	0	—	—	—	16.1	—	Water № 27 after centrifuging
28	8.XI	6.5	0	—	—	24.6	14.1	57	The same after treatment with $Al_2(SO_4)_3$
28	8.XI	6.5	5.5	12.1	127.00	24.2	14.2	58	—
32	9.XI	—	0	12.0	124.21	24.1	13.0	54	34 miles north-west of the village Kizyl-Kup
35	10.XI	6.5	0	11.9	90.19	21.3	13.8	65	Near the entrance to the Karabugaz strait
35	10.XI	6.5	6.0	12.5	123.63	24.2	15.8	65	

* For the determination 100 ml of sea water were taken for stations Nos 24—44, 25 ml of sea water + 75 ml of distilled water for station № 18, and 10 ml of sea water + 90 ml of distilled water for the Karabugaz Bay; 10 ml of a 0.01*n* $KMnO_4$ solution and 5.0 ml of a 20% NaOH solution was added (in the case of an alkaline medium); time of boiling, 10 minutes.

** In the column of salinity the chlorine content is given in ‰.

being formed]. The investigations have shown that the oxidability of water was 90 per cent of the original value subsequent to the first method of separation, 83 per cent after the second, and 76—83 per cent after the third. This is an evidence that the bulk of the organic matter of sea water is in a dissolved state.

5) The calculated percentage ratio of the neutral oxidability to the alkaline one ranges from 55 to 61 per cent, being on the average 57 per cent; this indicates that the composition of the organic matter of the sea water of the bay closely approaches to that in the open sea throughout the whole depth of the water.

6) In order to compare the oxidability of the given water with that from the Barentz Sea, station № 2149 was chosen (expedition of the «Persey», VIII, 1932) on the central line of Nordcape current of the Gulf Stream; depth, 320 m; the data of the neutral oxidability are as follows: 0 m—0.60 mg; 25 m—0.31 mg; 50 m—0.14 mg; 100 m—0.14 mg; 310 m—0.14 mg O_2([1]).

It will be seen that the content of organic matter in the water of the south-eastern part of the Caspian Sea is three—five times (seven—nine times) as large as that for the Barentz Sea at 0—25 m (at 50—300 m respectively); a similar relation (three—eight times) has been found also for albuminoid ammonia ([5,6]). Such a large accumulation of organic matter in the water of the Caspian Sea is a result of a heavy mainland-drainage (rivers), the given basin being quite closed.

7) In passing to the Karabugaz Bay we shall see that the average value of oxidability of its surface waters is 8.5 times that of the open sea value; the bottom value at station № 35 is still greater; the percentage ratio of the neutral oxidability to the alkaline one closely approaches to that of the sea water, which points to the similarity in their composition. Experiments on the separation of the organic matter made with the water of station № 28 have shown that practically the whole of its mass is in a dissolved state; at the same time these data indicate that the considerable amount of matter in suspension present in the water of the bay (which produced a 10 cm transparence in 1936) consists essentially of the mineral substances. The specific features of the hydrobiological and meteorological regime of the Karabugaz Bay being taken into consideration, the reason of such an accumulation of organic matter may be easily established. However, with regard to chlorine, the water of the bay in November 1936 had a nearly 23 times greater concentration than the water of the Caspian Sea. Thus, we have an obvious «loss» of organic matter by the water of the bay; this may be due, on the one hand, to its consumption by heterotrophic bacteria, the presence of which has been pointed out by A. D. Pelš ([8]); on the other hand, quite probable is a partial coagulation of the organic matter with its subsequent deposition on the bottom, where it serves as the source of feeding desulphating bacteria. In conclusion is should be noted that the observations made by the author have proved the possibility of using also under these conditions the permanganometric method with a iodometric end, after a preliminary tenfold dilution of the water.

B. Biochemical Consumption of Oxygen

The data given in Table 2 show that:

1) The decrease in the amount of oxygen due to biochemical processes, during a period of 120 hours, at 18—20° C, in the superficial water of the Krasnovodsk Bay and in the open part of sea, is 0.50—0.70 mg O_2, which corresponds to the biological consumption of oxygen in the purest artesian

Table 2

Results of Determination of Biochemical Oxygen Consumption of Sea Water at 18—20°C in mg O_2 per 1 liter

№ of station	Depth at which the sample was taken, in m	Biological consumption of oxygen		
		72 hours	120 hours	270 hours
	a) Krasnovodsk Bay			
24	0	—	0.68	0.92
24	1.2	—	0.73	1.24
	b) Open sea			
42	0	0.27	0.50	0.86
	c) Karabugaz Bay*			
28	0	—	1.20	—
28	5.5	—	2.25	—

waters, as it has been recorded by S. V. Bruevič for the Barentz sea (6). Closely approaching figures have been obtained by the author for the waters of the Kola Bay (2).

2) The biological consumption of oxygen in the water of the Karabugaz Bay is represented by a greater value. It is of interest to note that this value is near that of the complete saturation of the given water oxygen. The latter value, according to the author's experiments made at 17° C, amounts to 1.83—1.90 mg O_2 in 1 liter (i. e., 4.8 times less than in the Caspian Sea).

II. Sea Foam. Oxidability and Biological Consumption of Oxygen

Three samples of sea foam were collected: in the Krasnovodsk Bay, the Saimonov creek (station № 18), and in the Karabugaz Bay (stations Nos 27 and 28).

The first sample was collected immediately near the shore where the foam was formed when there was a strong stormy wind, it was collected by means of a spoon and placed in a wide jar. After the settling of the whole of the foam, the volume of the water obtained was equal to 750 ml; its colour was slightly yellowish; the turbidity, great, silt, detritus, and fragments of algae were in suspension, it frothed easily and had an algal odour; after 10 day standing a certain clearing was observed and no putrid odour was noted; when evaporated to dryness and dissolved in hot distilled water, it retained its foaming ability but became still clearer. Chemical properties: pH =8.10—8.30; alkalinity =2.98 m.-equiv. (water =2.73); Cl =6.75 g/l (water 6.28 g); amount of suspended matter after a 10 minutes centrifuging was 500 mg/l.

The second and third samples (the Karabugaz Bay) were collected directly from on board the ship, 15 miles north of the shore (village Kizyl-Kup) with a gauze sack. Here the foam was stable, and formed in the open part of the bay even when the wind was of medium strength. The colour was white, the odour pleasant probably due to algae.

* The water from the Karabugaz Bay was preliminarily diluted five times with the water from the Krasnovodsk Bay, for the biological oxygen consumption of which a correction was made; in the values given the dilution has been taken into consideration.

Table 3

Results of Determination of Oxydability and Biological Consumption of Oxygen in Sea Foam in mg O_2 per liter

№	№ of station	Oxidability*			Biological consumption of oxygen during 120 hours at 18—20°C**	Supplement
		Neutral				
		Alkaline	Absolute	In % of the alkaline		
1	18	123.0	—	—	11.0	—
2	18	111.0	—	—	—	№ 1 after centrifuging
3	18	19.0	—	—	—	№ 1 after treatment with $Al_2(SO_4)_3$
4	18	130.0	—	—	—	№ 1 after standing for 120 hours
5	18	130.0	80.0	61	—	№ 1 after evaporating to dryness, drying and dissolving in distilled water
6	27	72.0	41.0	57	—	№ 5 after centrifuging
7	27	78.0	39.0	50	—	—
8	27	81.0	35.0	43	—	№ 7 after centrifuging
9	27	—	19.0	—	—	№ 7 after treatment with $Al_2(SO_4)_3$
10	28	76.0	35.0	46	12.0	—

The results of investigation (Table 3) show that:

1) The oxidability of sea foam exceeds that of water nearly twenty times in shore, and about three times in the open bay. Such a phenomenon is quite natural if the conditions of foam formation at station № 18 (during a strong surf) be considered. The experiments made on the separation of the organic matter have shown that a treatment of the foam with $Al_2(SO_4)_3$ greatly decreases the oxidability; this fact indicates that the organic matter of the foam is essentially maked up of colloids. Keeping the foam during 120 hours, and evaporating it to dryness and drying practically do not change the value of oxidability; however, the latter treatment naturally lowers the oxidability of the centrifugate (as a result of coagulation of the colloid). Attention should be paid to the fact that two samples of foam collected in the Karabugaz Bay during the two following day (on the 7th and 8th of November) showed a constant oxidability. The ratio of the neutral to the alkaline oxidability is subject to marked variations.

* For the determination, 5 ml of foam + 95 ml of distilled water + 30 ml of a 0.01 *n* $KMnO_4$ were used.

** The foam was diluted 10 times with the water from the Krasnovodsk Bay; this was taken into consideration when calculating the biological consumption of oxygen.

2) The biological consumption of oxygen in the foam 15 or 10 times exceeds that in water.

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GENETICS

STUDIES ON POLYPLOID PLANTS. XVIII. CYTOGENETIC STUDIES ON *NICOTIANA SILVESTRIS* × *N. TOMENTOSIFORMIS* HYBRIDS AND AMPHIDIPLOIDS AND THEIR BEARINGS ON THE PROBLEM OF THE ORIGIN OF *N. TABACUM*

By DONTCHO KOSTOFF

(Communicated by N. I. Vavilov, Member of the Academy, 4. I. 1938)

The cultivated tobacco, *Nicotians tabacum* L. is a species which is very rarely found in nature, i. e. in «wilderness» and taxonomists are inclined to assume that such sporadically found plants are spread secondarily from those that are in cultivation. The question has been often discussed as to its origin. About 10 years ago, at the 5th International Congress of Genetics Clausen (1927/1928) (3) advanced the hypothesis that *N. tabacum* ($n=24$) has originated from a species hybrid between *N. silvestris* ($n=12$) and *N. tomentosa* ($n=12$). Clausen (4) built up his hypothesis on the basis of the following arguments: 1) *N. tabacum* has twice as many chromosomes as the presumed ancestral species, 2) *N. tabacum* haploid has meiosis like the F_1 *N. silvestris* × *tomentosa*, i. e. with asyndesis, or only one or a few bivalents being formed, 3) F_1 *N. tabacum* × *silvestris* and F_1 *N. tabacum* × × *tomentosa* form during the first meiotic metaphase 12 bivalents and 12 univalents. At the same Congress Brieger (1927/28) (1) rejected Clausen's hypothesis assuming that the genes from the presumed ancestral species are not present in *N. tabacum*.

In examining Clausen's hypothesis I attempted to cross *Nicotiana tabacum* with F_1 *N. silvestris* × *tomentosiformis*. *N. tomentosiformis* (called *N.* Rusbyi, $n=12$ in the earlier papers) is a species closely related with *N. tomentosa*, it crosses with *N. tomentosa* readily and their hybrids are fully fertile having normal meiosis. I preferred *N. tomentosiformis* to *N. tomentosa* because: 1) the hybrid F_1 *N. silvestris* × *N. tomentosiformis* are more *N. tabacum*-like morphologically and 2) *N. tomentosiformis* has a relatively shorter vegetation period than *N. tomentosa*, suggesting its closer relation with *N. tabacum* F_1 hybrids. *N. silvestris* × *tomentosiformis* is self sterile but it forms occasionally high percentage of non-reduced viable gametes especially when exposed to high temperature and unsufficient water supply. From the triple crossing *N. tabacum* × *(silvestris* × *tomentosiformis)* I obtained a large number of hybrids (Kostoff, 1930, 1931) (6,7). Most of them had $n=24$, $2n=48$ chromosomes and were self fertile having almost normal meiosis (Kostoff 1933)(8), while others had chromosome numbers different from 48 and were partially fertile. At the same time Brieger

(1930)([2]) independently obtained a similar triple hybrid, but he crossed F_1 (*N. tabacum* × *tomentosiformis*) × *N. silvestris*.

The fully fertile triple hybrids *N. tabacum* × (*silvestris* × *tomentosiformis* were the first experimental proof of Clausen's hypothesis with the correction that *N. tomentosiformis* rather than *N. tomentosa* seems to be the one of the ancestral species (Kostoff, 1933) ([8]). In the triple fully fertile hybrids the chromosomes of *N. silvestris* (n=12) conjugate with 12 chromosomes of *N. tabacum* (n=24) while the other 12 chromosomes of *N. tabacum* conjugate with *N. tomentosiformis* chromosomes, thus in the fully fertile triple hybrids 24 bivalents are formed which disjoin quite normally during the I anaphase. The partial homology of one or a few *tomentosiformis* and *silvestris* chromosomes led occasionally to formation of trivalent (rarely quadrivalent) chromosomes which further introduced some abnormalities in the meiosis as pointed out before ([8]). The chromosome constancy of these hybrids is high but they segregate in respect to their characters thus producing numerous forms, many of them reminding of the already existing varieties of *N. tabacum*.

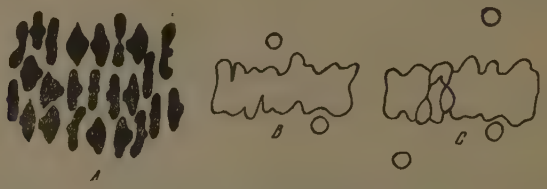

Fig. 1. *A*—24 bivalent chromosomes in the amphidiploid, *B*—side view of 1 metaphase plate with 2 univalent, and *C*—also a side view of first metaphase but with a trivalent and three univalent chromosomes in the amphidiploid *N. silvestris*—*tomentosiformis*.

The hybrids produced in crossing the fertile triple hybrids with *N. tabacum* were fertile too.

The cytogenetic behaviour of the triple hybrids having whole chromosome sets from *N. silvestris*, *tomentosiformis* and *tabacum* (12+12+24=48) represented the first experimental proof of Clausen's hypothesis. But it was necessary to supply one more evidence into this line, namely, it was necessary to produce the species *N. tobacum* synthetically in doubling the chromosome number in the hybrid *N. silvestris* × *N. tomentosiformis* (Kostoff, 1936) ([9]). This was a very difficult task, because F_1*N. silvestris* × *N. tomentosiformis* are sterile hybrids and because tobacco plants do not produce readily wound-callus shoots as tomato does among which tetraploids could be expected. During several years I insistently crossed F_1 *N. silvestris* × *tomentosiformis* with pollen of *N. silvestris*, and only a very small number of seeds were produced. From these seeds plants were raised among which two plants were found with 36 chromosomes, while the other plants had more or less than 36 chromosomes. These two plants with 36 chromosomes were triploids having two genoms *N. silvestris* and one from *N. tomentosiformis*. They had irregular meiosis and were sterile but they set seeds more readily in crossing them with *N. tomentosiformis*, than in crossing F_1 with *N. silvestris*. From the cross (*N. silvestris* × *tomentosiformis*) × *N. silvestris* × *tomentosiformis* a large amount of seeds was obtained, from which numerous plants were raised. The plants differed morphologically very greatly. I selected for cytological studies such forms that were in appearance like the F_1 hybrids. Among these plants I found two that had 48 chromosomes while the others had more than 48 chromosomes or less than that. These two plants were almost fully fertile and were very much *N.*

tabacum like. Morphologically they were not quite alike, especially in respect of the flowers they formed. Both were, however, allotetraploids *N. silvestris—tomentosiformis*. In crossing these two plants with *N. tabacum* we obtained fertile hybrids, which segregated further in the way distant varietal crosses of *N. tabacum* do segregate.

The hybrids with more or less than 48 chromosomes were partially fertile. In their progenies several types with 48 chromosomes were segregated as well as several types of chromosomal aberrants. Some of the former remind very much of certain segregates obtained in selfing the triple fully fertile hybrid *N. tabacum* × (*silvestris* × *tomentosiformis*).

In studying three subsequent generations obtained from both allotetraploid plants *N. silvestris—N. tomentosiformis* I found that there gradually appeared plants with greater fertility (i. e. fully fertile) or with lower fertility (i. e. partially fertile). The cause of the phenomena described above can be sought for in the chromosome behaviour in F_1 hybrids and in forms obtained by back-crosses.

Both allotetraploid plants produced progenies that were morphologically highly uniform; only a very low percentage appeared that difffered in some respects from the parental forms.

In studying the meiosis in the F_1 hybrids *N. silvestris* × *tomentosiformis* I found a very variable number of bivalent formations. External conditions seem to exert some influence on the formation of bivalents, since in one and the same plant I found in summer a larger percentage of bivalents than in autumn. The observations made in summer are given in the table.

Table

The Number of Bivalent Chromosomes Formed in the Pollen Mother Cells (P.M.C.) of the F_1 Hybrid *N. silvestris* × *N. tomentosiformis*

	Number of bivalents found in the P.M.C.						% of P.M.C. having at least one bivalent	Trivalents	Polyvalent
	0	1	2	3	4	5			
P.M.C.	35	23	16	9	3	1	61.2	2	1

The bivalents in F_1 hybrid are usually hold by one chiasma.

The data given in table show that 61.2% of the pollen mother cells (P.M.C.) have at least one bivalent, i. e. at least one chiasma per cell. In other words 52 out of 85 P.M.C. give rise to cross-over gametes. It is probable that the majority of cross-over gametes (may be even all of them) are viable judging from the viability of $n-1=23$ chromosomic gametes formed in the allopolyploid *silvestris—tomentosiformis* monosomics and in the monosomics *N. tabacum* (Clausen, 1932, ([5]), Olmo, 1936 ([10]) .

The formation of different gametes *with* somatic chromosome number following crossing over between *silvestris* and *tomentosiformis* chromosomes have obviously conditioned the differences found between the first two amphidiploids that were obtained.

Allopolyploids formed usually 24 bivalents but occasionally univalents and polyvalents were also observed, which accounts for further anomalies in meiotic processes and which explains the polymorphism of the species *N. tabacum*.

The above presented data confirmed Clausen's hypothesis of the origin of *N. tabacum*. It would be illogical to demand that our synthetically pro-

duced *N. tabacum* forms should be identical with certain varieties of *N. tabacum*, since the historical elements may not be put aside. It would be interesting to add, however, that most of the allopolyploid forms *N. silvestris—tomentosiformis* appear so much like *N. tabacum*, that they are classed by tobacco experts among the species *N. tabacum*.

The areas of geographical distributions of *N. tomentosiformis* and *silvestris* are contiguous and may overlap in North Argentina—South Bolivia, the most probable region where *N. tabacum* has originated chromosome doubling in the F_1 hybrids *silvestris—tomentosiformis*.

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GENETICS

HETEROCHROMATIC (INERT) REGIONS IN THE CHROMOSOMES OF *CREPIS CAPILLARIS*

By DONTCHO KOSTOFF

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Genetic investigations on the inert regions in *Drosophila* (Muller, 1918, Muller and Gershenson, 1935) in connection with the cytological studies on the heterochromatine (inert chromatine) and chromocenter (Heitz, 1928, 1935, Painter and Stone, 1935, Prokofieva, 1935, Frolova 1935, etc.) showed that the chromosomes have active (euchromatic) and inert (heterochromatic) segments, the proximal parts, near the centromeres, being heterochromatic. These regions unite in the chromocenter in the salivary gland nuclei.

The data reported by Sturtevant (1936) on the preferential segregation in triplo-IV female *Drosophila melanogaster* and those by Kikawa (1937), who studied haplo-IV *Drosophila ananasse*, indicating the probable conjugation between V and IV chromosomes, suggested that non-homologous chromosomes most probably conjugate in the inert regions. More convincing evidence in this respect was supplied by Gershenson in his work on the preferential segregation in triplo-IV flies having XY+fragmented X. Direct cytological evidence was supplied by Prokofieva (1937), who showed that the second (inert) arm of IV chromosomes conjugates with the inert region of the left arm of the III chromosome.

A series of cytogenetic phenomena that occur in plants can be best interpreted if one assumes a conjugation between the inert regions of the non-homologous chromosomes. Heitz (1929) and others showed that plant chromosomes (especially the sex chromosomes) have heterochromatic regions, but an exact analysis of the whole caryotype, as in *Drosophila* (Heitz, 1933, Prokofieva, 1935) has not yet been carried out, therefore we attempted to analyse the chromosomes of *Crepis capillaris*. This species has three distinct pairs of chromosomes: *A*, *C* and *D*. Haematoxyline-preparations with completely black chromosomes were slightly distained (fixation Lewitzky chromic acid-formalin). The proximal regions in each pair and the very distal ends remained darker, while the intermediate regions became lighter (figure). It should be noted here that the heterochromatic regions were not always exactly equal in length in all preparations, as shown in the figure. In some preparations the heterochromatine of the distal region (end) of *C* and *D* chromosomes were hardly recognizable. We cannot decide now whether in these particular cases it was completely lacking, or whether the chromosomes were highly distained. It is probable that the slight variations in length of the heterochromatic regions are due to the difference in density

of euchromatine and heterochromatine. Such regions may be partially active. In studying the karyotypes of *Secale* species (Kostoff, Dogadkina and Tikhonova, 1935) and *Nicotiana* species, euchromatic and chromatic regions have also been found (compare chromosome of *Secale Vavilovi*).

The above reported data (also some others that cannot be recalled here on account of the lack of space) in connection with the data reported by other investigators, a part of which were mentioned above, demand a revision of some cytogenetic conceptions and suggest new interpretations of known phenomena. We shall mention here a few of them, because we think that our hypothetical assumptions will stimulate research work.

McClintok's (1933) data, for example, upon the non-homologous chromosome conjugations in *Zea* can be explained assuming that a conjugation between inert regions takes place. «Fold backs» of the univalents are due to the attraction of inert regions. The data of this author as well as Avdulov's data (1933) showed that in *Zea* even entirely inert chromosomes occur. Terminal conjugation of non-homologous chromosomes in haploids, as for example in *Triticum monococcum* ($n=7$) (Kihara and Katayama) is obviously due to conjugation of distal heterochromatic regions of non-homologous chromosomes. Numerous «mutual» translocations in *Crepis* reported by many authors can be interpreted by the assumption of somatic crossing over in conjugating inert regions. This concerns only the dinamics of the phenomenon. Stern (1936) supplied convincing data as to the occurrence of somatic crossing over, and Peto (1935) (in plant somatic tissue) and Levan (1937) (in pollen) found direct cytological evidences.

Demerec and Hoover (1936) found a viable deficiency in *Drosophila*. They wrote: «The deficiency 260—5 does not involve any of the visible extant loci. It takes a section of at least four bands at the tip end of the *X*-chromosome. Flies homozygous for this deficiency are viable and fertile. They do not show any visible peculiarities». This deficiency is viable because it probably affects only the inert region. When one examines the draw-

ings by Prokofieva (1935a, Fig. 1, 4, 5 and 6), one sees that the very distal end of the *X*-chromosome is inert.

In the light of these studies we should endeavour to find out in how far the catenation in *Oenothera* is due to chromosome homology of the distal regions or to the conjugation between inert regions of the distal ends. *Oenothera* «mutations» which result from structural changes might as well be due to crossing over in the inert regions of non-homologous chromosomes.

The probable rôle of the inert regions in the interspecific hybridization and in the evolution will be discussed elsewhere.

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GENETICS

COMPARATIVE STUDY OF CELL SIZES OF AUTO- AND ALLOPOLYPLOIDS

By A. I. ZHURBIN

(Communicated by N. I. Vavilov, Member of the Academy, 2. XII. 1937)

This research aimed to compare the effect of the increase of the chromatine mass and the influence of genoms of genetically close and remote parents on the size of the plant cell. Autopolyploids: diploid, triploid and tetraploid *Nicotiana glauca* have been taken as plants which have cells, containing different quantities of chromatine mass. Allotetraploids: amphidiploid *Nicotiana multivalvis* × *Nicotiana suaveolens* and amphidiploid *Secale montanum* × *Tricicum durum* have been taken as plants which have cells, containing genoms more or less remote. The initial forms have also been taken. All the plants have been grown under analogous conditions. The measuring of stomata, i. e. of the two closing cells taken together has been carried out on fully developed leaves fixed in 70° alcohol by means of an ocular micrometer. The middle of the leaf lamina near the main vein has been taken for these measurings. The results of the measurements are given in the table.

The area of stomata, calculated approximately by means of multiplying the length by the width, are given at the foot of the first (from right side) column.

The examination of the data obtained on di-, tri- and tetraploid *N. glauca* show that in fact the increase of the cell sizes depends on the increase of chromatine mass (the number of chromosomes). The increase of the chromatine mass proceeds in arithmetical progression with a ratio equal to 12 (12, 36 and 48 chromosomes). The increase of the cell area approaches the geometrical progression wiht a ratio 1.2 (862.84, 1051.22 and 1332.63 sq. microns).

These data coincide with the observations of Sinott, Houghtaling and Blakeslee (1934) (4).

The examination of the data on the size of the cells of the interspecific and intergeneric amphidiploids and their parent forms shows that in the first case the area of the cells has approximately increased 1.45 times: the area of the parent cells being 797.76 and 1137.28 sq. microns and that of the amphidiploid cells, 1410.71 sq. microns; in the second case the increase was 1.82 times: in parents 1400.13 and 1543.80 sq. microns, and in amphidiploid 2684.42 sq. microns. Consequently, the increase of the area of the intergeneric amphidiploid cell is greater than that of the interspecific amphidiploid. Though the given forms seem to us uncomparable as to the sign of genetical remoteness of the genoms which have met in these amphidiploids

Plant	Chromosome-number (2n)	Lenght in microns: 18.9—22.0	22.1—25.2	25.3—28.4	28.5—31.6	31.7—34.8	34.9—38.0	38.1—41.2	41.3—44.4	44.5—47.6	47.7—50.8	50.9—54.0	54.1—57.2	57.3—60.4	60.5—63.6	63.7—66.8	66.9—70.0	70.1—73.2	73.3—76.4	76.5—79.6	79.7—82.8	82.9—86.0	86.1—89.2	89.3—92.4	92.5—95.6	95.7—98.8	n	$M \pm m$ (μ)	P (%)	σ (μ)	v (%)
Nicotiana glauca (2n)	24	—	1	27	66	86	44	26	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	250	33.25±0.23	0.66	3.65	11.00
» *glauca (3n)*	36	—	—	—	10	48	102	65	24	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	250	37.41±0.19	0.51	3.17	8.47
» *glauca (4n)*	48	—	—	—	—	1	19	59	68	74	26	3	—	—	—	—	—	—	—	—	—	—	—	—	—	250	43.65±0.22	0.51	3.74	8.57
» *multivalvis*	48	3	8	39	63	88	46	3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	250	32.00±0.22	0.69	3.71	11.59
» *suaveolens*	32	—	1	9	14	49	48	30	51	26	21	1	—	—	—	—	—	—	—	—	—	—	—	—	—	250	39.23±0.38	0.97	6.02	15.34
N. multivalvis×N. suaveolens	80	—	—	—	3	12	28	48	61	44	46	8	—	—	—	—	—	—	—	—	—	—	—	—	—	250	43.30±0.32	0.74	5.02	11.59
Secale montanum	14	—	—	—	—	—	2	2	13	14	68	75	60	16	—	—	—	—	—	—	—	—	—	—	—	250	52.03±0.27	0.52	4.22	8.11
Triticum durum	28	—	—	—	—	—	—	2	7	19	47	65	56	39	13	2	—	—	—	—	—	—	—	—	—	250	53.66±0.30	0.56	4.80	8.95
S. montanum×T. durum	42	—	—	—	—	—	—	—	—	—	—	3	3	9	12	7	9	15	29	51	42	24	22	11	6	7	250	78.40±0.60	0.77	9.41	12.00

Plant	Chromosome-number (2n)	Width in microns: 12.5—15.6	15.7—18.8	18.9—22.0	22.1—25.2	25.3—28.4	28.5—31.6	31.7—34.8	34.9—38.0	38.1—44.3	41.4—44.6	44.7—47.8	n	$M \pm m$ (μ)	P (%)	σ (μ)	v (%)	$M_l \times M_w$ (μ²)
Nicotiana glauca (2n)	24	—	—	11	114	89	33	3	—	—	—	—	250	25.95±0.16	0.62	2.59	9.98	862.84
» *glauca (3n)*	36	—	—	—	15	154	77	4	—	—	—	—	250	28.10±0.04	0.14	0.59	2.10	1 051.22
» *glauca (4n)*	48	—	—	—	1	59	122	65	3	—	—	—	250	30.53±0.05	0.16	0.75	2.46	1 332.63
» *multivalvis*	48	—	1	41	118	84	16	—	—	—	—	—	250	24.93±0.05	0.20	0.83	3.33	797.76
» *suaveolens*	32	—	—	4	28	87	88	40	3	—	—	—	250	28.99±0.06	0.21	0.98	3.38	1 137.28
N. multivalvis×N. suaveolens	80	—	—	2	3	28	59	109	47	2	—	—	250	32.58±0.06	0.18	1.01	3.10	1 410.72
Secale montanum	14	2	1	26	70	71	57	16	7	—	—	—	250	26.91±0.08	0.30	1.26	4.68	1 400.13
Triticum durum	28	—	—	—	23	106	98	22	1	—	—	—	250	28.77±0.05	0.17	0.80	2.78	1 543.80
S. montanum×T. durum	42	—	—	—	—	15	40	108	61	22	2	2	250	34.24±0.07	0.20	1.08	3.15	2 684.42

since they belong not only to different families but even to different subclasses of monocotyledonous and bicotyledonous plants.

In comparing the increase of autotetraploid *N. glauca* cells with those of allotetraploid *N. multivalvis* × *N. suaveolens* we find that in both cases this increase is the same, i. e. 1.5 times (for the autotetraploid 1332.63 : $\frac{862.84 + 862.84}{2}$ and for allotetraploid 1410.71 : $\frac{797.76 + 1137.28}{2}$). Though we have an equal increase of chromatine mass (the number of chromosomes) in both cases, the genetic remoteness of genoms is different: in the first case, the doubling of one and the same diploid set of chromosomes and, in the second case, two sets belonging to different species. Consequently, the genetical remoteness of the chromosome sets of the allotetraploid had no effect on the increase of the cells.

The phenomenon of vigour, observed in many F_1-hybrids, autopolyploids and allopolyploids is produced by the number of cells and their size. Facts are known when heterosis developed only as a result of a greater number, i. e. greater fission intensity of the cells, their size being smaller, compared to the parent cells, or an average one (smaller than in one parent and larger than in the other); for instance, in *N. tabacum*×*N. silvestris* [Kostoff, 1936, p. 430—435 (3)]. Facts are known when the hybrid compared to its parents possessed larger cells, though it was a dwarf. For instance, *N. paniculata*×*N. glutinosa*, *N. paniculata* × *N. tabacum* [ibid. p. 434—435(3)].

Consequently, the increase of the cell size does not always lead to the heterosis as the intensive fission of cell, i. e. their greater number, is necessary for this. The same can be said about the univalent and bivalent forms. For instance, in the bivalent form of the moss *Hyphum (accrocladium) cuspidatum* (L.) Lindb. the width of the leaf cell increased in comparison with that of the univalent form and the leaf width became narrower as the number of cells decreased [Wettstein, 1924, p. 176 (5)]; in the bivalent form of the moss *Funaria hygrometrica* (L.) Sibth. the size of the cells increased two times in comparison with the univalent form and the size of the leaves increased while the number of the cells (in the width of the leaf) remained the same [ibid, p. 59 (5)].

The explanation of heterosis as due to interaction of gens and the influence of the chromatine mass are well known. The investigators attach different importance to each of these explanations.

It is to be regretted that up to now the amphidiploids such as *N. multivalvis*×*N. suaveolens* and *S. montanum* × *T. durum* and polyploids *N. glauca* have been received in a very small number of specimens. This did not allow to determine their average growth. Therefore we have not yet the possibility to compare the chromatine mass and the genetic remoteness of genoms with the general increase of the plant and cells in the mentioned polyploids and their initial forms. We only know that amphidiploids *N. multivalvis* × *N. suaveolens* «proved to be more vigorous than F_1-hybrids and had somewhat larger flowers» [Kostoff, 1937 (2)].

Besides this the amphidiploid had larger cells than F_1 and its parents and the diameter of its pollen grains had 19.56 points of the ocular micrometer, while the pollen of *N. multivalvis* had 15.2 points in diameter; and *N. suaveolens*, 15.5 points of the ocular micrometer (1). In comparing these data about pollen with our data about the stomata sizes, we find that the pollen grains as well as the stomata of the amphidiploid have increased to the same extent as the parent species.

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GENETIK

EINE METHODE DER SAMENGEWINNUNG BEI SELBSTBESTÄUBUNG AUTOSTERILER RÜBENSORTEN

Von E. HARETSCHKO-SAWIZKAJA

(Vorgelegt von N. I. Vavilov, Mitglied der Akademie, d. 21. I. 1938)

Die Autosterilität ist eine charakteristische Eigenschaft der Art *Beta vulgaris*. Autofertile Rübensorten sind ausserordentlich selten. Die Ursachen für die Autosterilität der Rüben liegen, wie durch unsere Untersuchungen ([15],[16]) festgestellt worden ist, 1) in dem verzögerten Wachstum der Pollenschläuche und 2) in der Degeneration der befruchteten Zygoten. Der Hauptfaktor der Autosterilität ist das langsame Wachstumstempo der Pollenschläuche bei Selbstbestäubung und das Fehlen von Befruchtung. Der Befruchtungsprozess bei Selbstbestäubung und Isolierung wurde an vielen Rübensorten untersucht. Von 3953 Samenknospen erwiesen sich nur 87, oder 2.2%, als befruchtet. Die ungeheure Mehrzahl der Pflanzen ist während der gewöhnlichen Blüteperiode autosteril und eine Selbstbefruchtung bleibt aus. Die autofertilen Rübensorten sind charakterisiert durch das schnelle Wachstum der Pollenschläuche und einen hohen Prozentsatz an Selbstbefruchtung. Die Autosterilität der Rüben ist also auf die Ursachen zurückzuführen welche für die Autosterilität fremdbestäubter Arten festgestellt worden sind [East und Park ([6]) bei *Nicotiana*; Knight ([9]) und Sansome ([13]) beim Apfelbaum, Moore ([12]); Andersen und Sax ([1]) bei *Tradescantia*; Asami ([2]) beim Birnbaum; Crane und Lawrence ([5]) bei Pflaumen- und Kirschbäumen u. a.]. Das langsame Wachstum der Pollenschläuche bei Selbstbestäubung autosteriler Sorten, wird, wie Jassuda ([14]) bei den Petunien festgestellt hat, durch Anhäufung in den Pistillgeweben von Stoffen, die das Wachstum der Pollenschläuche der gleichen vegetativen Linie verzögern.

Der Grad der Autosterilität variiert mit der Veränderung des Milieus. Bekanntlich sind viele autosterile Pflanzen gegen Ende der Vegetationsperiode [East ([7])], Jassuda ([14])] teilweise fertil. Korzhewin ([10]) erhielt Samen von der Selbstbestäubung von Veronica bei Hinzugabe von Magnesiumsalzen zum Boden. Myers ([11]) weist darauf hin, dass bei Verschlechterung der mineralischen Ernährungsbedingungen die autosterilen Pflanzen *Brassica pekinensis* die Fähigkeit zur Samenbildung erlangen. East([7]) erhielt Samen von den autosterilen Pflanzen *Nicotiana* bei der Selbstbestäubung junger Knospen einige Tage vor ihrem Aufknospen. Diese Methode wurde mit Erfolg von Kakizaki ([8]) bei Kohl angewandt. Buchholz und Doak ([3]) erhielten Samen bei der Selbstbestäubung von Datura durch Transplantation der Griffel. Die Anwendung dieser Methoden ermöglicht eine Nachkommenschaft durch Selbstbestäubung von Formen, die bei gewöhnlichen Bedin-

Keimung der Knäulchen in der kalten Kammer

Mittlere Temperatur (Celsiusgrade). In Intervallen von 10 Tagen	Die Pflanzen wurden die ganze Zeit in der kalten Kammer gezogen					Pflanzen, die aus der warmen Kammer am 10.III. hinübergebracht wurden		
	Pfl. 1	Pfl. 3	Pfl. 4	Pfl. 5	Pfl. 6	Pfl. 7	Pfl. 12	Pfl. 13
1.I.—10.I., 11.8°	—	—	—	—	—	—	—	—
11.I.—20.I., 11.8°	—	—	—	—	—	—	—	—
21.I.—31.I., 11.8°	—	—	—	—	—	—	—	—
1.II.—10.II., 11.7°	—	—	—	—	—	—	—	—
11.II—20.II., 10.7°	—	—	—	—	—	—	—	—
21.II.—28.II., 10.0°	—	—	—	—	Isolierung 100%	—	—	—
1.III.—10.III., 10.0°	—	Isolierung 95.0%	Isolierung 90.5%	—	Bestäub. innerhalb d. Klones 78.8%	Isolierung 79.2%	Isolierung 52.3%	Isolierung 58.1%
11.III.—20.III., 13.3°	Isolierung 79.5%	Bestäub. innerhalb d. Klones 72.9%	Bestäub. innerhalb d. Klones 76.5%	Bestäub. innerhalb d. Klones 65.0%	—	Bestäub. innerhalb d. Klones 76.3	Bestäub. innerhalb d. Klones 74.5%	Bestäub. innerhalb d. Klones 26.4%
21.III.—31.III., 13.8°								
1.IV.—10.IV., 14.0°	—	—	—	—	—	—	—	—
11.IV.—20.IV., 13.1°	—	—	—	—	—	—	—	—
21.IV.—30.IV., 13.5°	—	—	—	—	—	—	—	—

gungen autosteril sind, zu erhalten. Jedoch stösst eine grosszügige Anwendung dieser Methoden auf eine Reihe von Schwierigkeiten. Die Methode der Transplantation der Griffel kann wegen ihrer Kompliziertheit nicht in grossem Masstabe verwendet werden. Hingegen ist die Methode der Bestäubung junger Knospen bei Kulturen mit kleinen Knospen und wenig Samen in einer Blüte unbequem.

Auf der Suche nach einer künstlichen Selbstbestäubungsmethode autosteriler Rübensorten waren wir bestrebt solche Bedingungen aufzufinden, bei welchen sich diejenigen genetischen Faktoren nicht entwickeln konnten, die die Autosterilität bedingen. Die Untersuchungen wurden an bei gewöhnlichen Bedingungen völlig autosterilen, sich vegetativ vermehrenden Klonen der Zuckerrübe durchgeführt. Die Pflanzen dieses Klons wurden im Winter in einer Orangerie kultiviert. Alle Pflanzen wurden gemeinsam bei 12—13°C bis zum Beginn der Knospenentwicklung aufgezogen. Dann wurden sie in zwei isolierte Kammern mit verschiedener Temperatur gebracht. Die Gewächse 3, 4, 5, 6 und 7 wurden die ganze Zeit in der Kammer mit niedriger Temperatur kultiviert. Die Pflanze 1 blühte erst in der kalten Kammer und wurde dann am 22.III. in die warme Kammer überführt. Die Pflanze 2 trug durch die zu niedrige Temperatur in der kalten Kammer einen Schaden davon, worauf sie am 18.II. innerhalb der Klonsgrenzen

in die warme Kammer gebracht wurde. Alle beschädigten Knospen dieser Pflanze kamen um; in der warmen Kammer bildeten sich neue Zweige und keimten Samen (Fig. 1). Die Pflanzen 12 und 13 wurden erst in der warmen Kammer gezogen und am 10. III. in die kalte Kammer gebracht. Alle in den isolierten Kämmern blühenden Pflanzen bestäubten sich frei innerhalb der Klonsgrenzen. Bei einem Teil der Pflanzen wurden einzelne Zweige durch Pergamentisolatoren zwecks Bestäubung innerhalb der Grenzen der Pflanze isoliert. Der Prozentsatz der Selbstbefruchtung wurde durch Zählung der gekeimten und nicht gekeimten Knäulchen bestimmt. Der Prozentsatz der bei verschiedenen Temperaturen gekeimten Knäulchen ist in Tabellen dargestellt. (Die geschweiften Klammern in den Tabellen weisen auf die Blüteperiode jeder Pflanze hin.)

Die angeführten Resultate weisen darauf hin, dass die Temperatur einen ungeheuren Einfluss auf die Entwicklung der Autofertilität hat. Bei niedrigen Temperaturen werden autosterile Pflanzen autofertil. Offenbar wird bei tiefen Temperaturen die Anhäufung in den Pistillen von Stoffen, die das Wachstum der Pollenschläuche und die Entwicklung der Autosterilität aufhalten, verhindert. Am wirksamsten erwies sich die Temperatur von ungefähr 10°C, bei welcher die Keimung der Knäulchen von 90—100% schwankte. Bei 13—14°C schwankt der Prozentsatz

Keimung der Knäulchen in der warmen Kammer

Mittlere Temperatur (Celsiusgrade) in Interwallen von 10 Tagen	Pflanzen, die aus der kalten Kammer hinübergebracht wurden		Die Pflanzen wurden die ganze Zeit in der warmen Kammer gezogen			
	22.III	18.II	Pfl. 8	Pfl. 9	Pfl. 10	Pfl. 11
	Pfl. 1	Pfl. 2				
1.I.—10.I., 12.5°	—	—	—	—	—	—
11.I.—20.I., 15.2°	—	—	—	—	—	—
21.I.—31.I., 15.5°	—	—	—	—	—	—
1.II.—10.II., 15.7°	—	Bestäub. innerhalb d. Klones 38.3%	—	—	—	—
11.II—20.II., 18.3°	—		Isolierung 20.8% Bestäub. innerhalb d. Klones 29.6%	Isolierung 21.0% Bestäub. innerhalb d. Klones 15.3%	Isolierung 18.4% Bestäub. innerhalb d. Klones 18.0%	—
21.II.—28.II., 22.8°	—					—
1.III.—10.III., 23.5°	Bestäub. innerhalb d. Klones 11.2%	—	—	—	—	Bestäub. innerhalb d. Klones 4.9%
11.III.—20.III., 25.2°		—	—	—	—	
21.III.—31.III., 25.0°		—	—	—	—	
1.IV.—10.IV., 23.0°		—	—	—	—	

der bei den einzelnen Pflanzen zur Keimung gelangenden Knäulchen von 65—79.5% und beträgt im Mittel 75.4%.

Bei 18—23°C nimmt die Menge der keimenden Knäulchen ab und schwankt zwischen 15.3 und 29.6% und beträgt im Mittel 20.5%. Bei Erhöhung der Temperatur auf 25°C fiel der Keimungsprozentsatz auf 4.9%. Die Pflanze 1, die bei 13—14°C 79.2% der Knäulchen zur Keimung brachte, senkte nach Überführung auf 24°C den Keimungsprozentsatz auf 11.2%. Die Pflanzen 12 und 13, die vor der Blüte eine verhältnissmässig hohe Temperatur (bis zu 23.5°C) genossen und dann für die Blütezeit in 13—14°C über-

Fig. 1—Die Zweige mit Knospen bei zu niedrige Temperatur beschädigt und mit Knäulen bei 15°C angesetzt.

geführt wurden, brachten beträchtlich weniger Samen zur Keimung (im Mittel 46.0%) als die Pflanzen, die vor der Blüte bei niedriger Temperatur aufgezogen wurden und bei der gleichen Temperatur von 13—14°C (75.4%) blühten. Offenbar begünstigt die Kultivierung von Setzligen bei hoher Temperatur in der Blüte voraufgehenden Periode in den Pistillen die Anhäufung von Stoffen, die das Wachstum der Pollenschläuche aufhalten. Bestäubung innerhalb des Klons und Bestäubung innerhalb der Grenzen der Pflanze (mit Isolator) gibt die gleichen Resultate.

Die mittlere Feuchtigkeit der Luft schwankte in beiden Kammern von 52—100% und betrug im Mittel 75—80%. Möglicherweise erleichtert grosse Feuchtigkeit ebenfalls die Selbstbefruchtung, jedoch gehört im gegebenen Falle der Temperatur die entscheidende Rolle. Eine Temperaturerhöhung verringerte auch bei grosser Feuchtigkeit den Prozentsatz der Samenkeimung.

Die erhaltenen Resultate wurden an Pflanzen, die aus Populationen genommen wurden, geprüft. Setzlinge, die vor der Blüte bei 12—13°C in einem Winterhaus gezogen wurden, dann mit Pergament isoliert und im April hinausgepflanzt wurden, gaben einen hohen Prozentsatz an gekeimten Samen. Eine Pflanze, die vor der Blüte bei 25—30°C gezogen und gleichzeitig ins Freie gebracht wurde, blieb völlig autosteril.

Die Aufklärung der Bedingungen, die die Entwicklung der Autosterilität-Autofertilität bei den autosterilen Pflanzen regulieren, gestattet uns eine Methode zur Gewinnung von Samen bei der Selbstbestäubung autosteriler Rübensorten vorzuschlagen.

Zur Gewinnung von Samen von der Selbstbestäubung dieser Formen müssen die Pflanzen in der Periode der Knospenentwicklung und der Blüte bei 10—13°C aufgezogen werden. Wahrscheinlich schwanken die einzelnen Genotypen in bezug auf die Temperaturen, die zur Samengewinnung nötig sind, nichtsdestoweniger sichert die Anwendung niedriger Temperaturen einen bedeutenden Prozentsatz an Samenkeimung. Die Verhältnissmässige Einfachheit dieser Methode, die weder künstliche Bestäubung noch irgendwelche komplizierten Behandlungen nötig macht, ermöglicht deren breite Verwendung in der Genetik und der Selektion der Rüben.

Die Analogie der Prozente, die die Autosterilität der verschiedenen Arten bedingen,—die Zunahme der Autofertilität bei vielen Kulturen gegen

Fig. 2—Der isolierte Zweig hat gesetzt bei 10°C—100% Knäulen; die anderen Zweige haben gesetzt bei 13—14°C—78.8% Knäulen.

Ende der Vegetationsperiode und die Beobachtungen von Jassuda bei den Petunien über die geringere Anhäufung von Stoffen, die das Wachstum der Pollenchläuche bei niedriger Temperaturen aufhalten, führen uns zu der Annahme, dass eine Temperaturerniedrigung die Entwicklung der Autosterilität auch bei anderen Kulturen behindern muss. Die Temperaturen, bei welchen die autosterilen Pflanzen die Möglichkeit zur Selbstbefruchtung erlangen, werden für die verschiedenen Arten verschieden sein. Dieselben sind an den Grenzen derjenigen Minimaltemperaturen zu suchen, bei welchen die Blüte der betrachteten Art möglich wird.

Die Schwankung der Temperaturen ist ein Faktor, der die Modifizierung der Autosterilität—Autofertilität bei der natürlichen Blüte bedingt. Dieselben Pflanzen können in Abhängigkeit vom Wettereinfluss verschiedene Grade der Autofertilität entwickeln. Dies muss bei Rübenkulturen in verschiedenen Klimaten und Gebieten in Betracht gezogen werden. In der Ukraine sind plötzliche Temperaturstürze während der Blüteperiode der Rübe selten und kurzzeitig und können keinen wesentlichen Einfluss auf eine Änderung

des Charakters der Fruchttragung haben. Aber bei den Setzlingskulturen im Norden und an den Küstengebieten, die durch grosse Feuchtigkeit und tiefe Temperatur charakteristisch sind, ruft ein bedeutender Prozentsatz an Selbstbefruchtung schroffe Veränderungen in der genetischen Zusammensetzung der Population hervor.

Ein spezifischer Unterschied der autofertilen Sorten von den autosterilen ist der verschiedene Bedarf am Temperaturregime für die Selbstbefruchtung. Die autofertilen Sorten, die keine Autosterilität bedingende genetische Faktoren besitzen und keine Stoffe anhäufen, die das Wachstum der Pollenschläuche hemmen, sind auch bei hoher Temperatur zur Selbstbefruchtung befähigt.

Der Einfluss der Temperaturbedingungen auf die Entwicklung der Autofertilität muss sowohl bei der genetischen und physiologischen Erforschung der Autosterilität—Autofertilität, sowie auch bei Ausarbeitung aller möglichen Methoden zur Gewinnung von Samen von autosterilen Rassen verschiedenen Alters und Zustands der Pflanzen, die auf Verwendung verschiedener Salze beruhen, berücksichtigt werden. Die Möglichkeit der Samengewinnung bei der Selbstbestäubung autosteriler Sorten eröffnet neue Möglichkeiten in der Genetik und Selektion der Rüben, indem sie die Untersuchung der genetischen Pflanzenstruktur an linearem Material gestattet und das Problem der Inzucht von einer neuen Ebene ausstellt. Die Inzuchtmethode kann mit Erfolg in der Selektion zur Aussonderung von Formen mit wertvollen rezessiven Eigenschaften (Immunität, Einblütler usw.) und zur Erziehung hochproduktiven Sorten durch Hybridisierung einzelnen Linien verwendet werden. Nach einer Mitteilung Coons[4]) gibt eine Hybridisierung der Inzuchtlinien der Rübe in den Vereinigten Staaten im Vergleich zu den handelsüblichen Sorten einen im Mittel um 39.3% höhern Ertrag. In unseren Verhältnissen sind die Chansen zur Erzielung ähnlicher Resultate bedeutend geringer weil die Rübe autosteril ist und sich die Arbeit mit der Inzuchtsmethode ausschliesslich auf die autosterilen Sorten konzentriert. Letztere stellen hinsichtlich der genetischen Mannigfaltigkeit ein äusserst beschränktes Material dar, das dank der Autofertilität in bedeutendem Masse zur Selbstbefruchtung und nicht zur Hybridisierung neigt. Die Anwendung der Inzuchtsmethode auf genetisch verschiedenartige autosterile Formen wird die Gewinnung ertragreicher Rübensorten ermöglichen.

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AGROPHYSIOLOGY

SOIL REACTIONS IN THE FORESTS OF LENKORAN AND WESTERN GEORGIA VIEWED UNDER THE ASPECT OF CITRUS AND TEA PLANT CULTIVATION

By A. I. POTAPOV

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In the search for new areas of tea and citrus plantations attention should be given to the study of the Lenkoran region, which in some climatical characters approachs the typical humid subtropics.

As the reaction of the soil is to be reckoned among the possible obstacles for tea cultivation, several specimens of soils from under various kinds of Lenkoran woodlands were taken by the author, and similar samples too from the tea region of Western Georgia, in the vicinity of Anasseuli. A comparative analysis of these two soil groups, as to the various forms of acidity, is given in Tables 1 and 2.

Table 1

Azerbaijan, Lenkoran Region

Kind of woodland	Relief	pH	Exchange acidity	Hydrolytic acidity	Notes
Albizzia, Parrotia, Carpinus	Table-land deprived of grass	6.8	0.3	9.2	In each case results of first titration are given. Determination according to Kappen and Deykuhara
Periploca, Parrotia	Valley	6.8	0.3	7.7	
Old oak (*Quercus castaneifolia*)	Hillock top	6.6	0.5	4.2	
Young oak	Table-land scarcely covered with grass	6.4	0.5	10.0	
Zelcova, Parrotia, Carpinus	Lowland, near Alekseievka	6.2	12.5	84.0	
Tea plantation, weeds	Girkanov tract, lowland	6.4	3.75	91.0	
No trees	Lowland	6.4	14.0	95.0	
Tea plantation	Lowland (near Lenkoran)	6.4	18.5	106.0	

The data of the tables show that the two countries are completely different in both physico-chemical and botanical characteristics of their soils.

They differ not only in the kinds of their woods but also in herbal vegetation. Two herbaceous plants have been noted by us as dependable indicators: fern and annual elder (*Sambucus ebulus*). In Western Georgia the fern thoroughly covers areas cleared of woods and the glades, and only in some

Table 2

Western Georgia, Ozurgeti Region

Kind of woodland	Relief and locality	pH	Exchange acidity	Hydrolitic acidity	Notes
Beech without undergrowth	Valley, Zvani	4.6	25.0	87.0	In each case results of first titration are given. Determination according to Kappen and Deykuhara
Beech, rhododendron	Anasseuli, hillock	4.3	61.0	108.0	
Ditto	Ditto	4.2	39.0	90.5	
Oak, hornbeam, camelia	Hillock	4.4	68.5	67.5	
Fern, bramble, acid grasses	Close to above, trees felled	4.4	35.0	82.5	
Fern, sod	Close to above	4.4	31.5	79.5	
Thick rhododendron	Hillock	4.4	48.0	74.5	
Hornbeam	Next to above	4.4	43.0	82.5	
Tea plantation, Anasseuli	Hillock, plantings	4.4	32.5	95.0	
Ditto	Ditto	4.4	36.5	72.5	
Ditto	Ditto	4.4	17.5	23.5	

places brakes of annual elder are met with. The latter always indicate the presence of marl beds, while fern is typical for soil poor in calcium.

Opposite are the relations observed in the Lenkoran region. Fern is but rarely found on the littoral hillocks or in the valleys, whereas elder forms extensive brakes in some places. Rhododendrons, so abundant in Western Georgia, are also absent here.

The analytical tables are in full agreement with the character of the vegetation in the regions examined. The neutral saturated soils of the Lenkoran littoral have their own vegetation which does not occur in Western Georgia (*Albizzia*, *Parrotia*, *Periploca*, *Zelcova*). The acid unsaturated soils of the southern part of the Black-Sea coast are characterized by the following vegetation: beech, hornbeam, bramble and fern.

As regards the inclosed valleys adjacent to the coastal region, such as the Vago valley in the Lenkoran region, we meet there, as seen from Table 1, soils with a high exchange and hydrolytic acidity, i. e. closely related as to their characters to the tea soils of Western Georgia. Yet a more thorough study is required, as the presence of pH =6.4 breaks the likeness and witnesses the specificity of Lenkoran soils. The majority of tea soils of Java have pH =5.5. Though soils with pH =6.8 also give a good growth of the tea tree, but diseases of the root system do already occur. Still greater are the requirements of Indian tea tree; even a reaction of soil with pH =6.0 unfavourably affects the growth of the tea tree in this country. In China the tea tree develops well on soil exceedingly poor in calcium (0.20 p. c.) and in magnesium (0.22 p. c.).

From this brief characteristic we may infer that the soils of the Lenkoran littoral region are more suitable for citrus than for tea trees, contrary to the soils of Western Georgia (Ozurgeti region). To utilize each of these regions for both kinds of crops it is necessary to change the reaction of Lenkoran and Ozurgeti soils according to the requirement of above plants.

To reduce the acidity and to increase the calcium content of the acid soils in Western Georgia for citrus plants is of course a much easier task than the shifting of pH in Lenkoran soils towards a greater acidity suitable for the tea-plants. In the latter case it is more expedient to explore the inclosed valleys adjacent to the littoral. As the precipitation is higher in the mountain region we may find there a more humid climate and more acid soils.

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TAXONOMY

A NEW VALUABLE RUBBER PLANT FROM THE PAMIRO-ALAI

By C. S. AFANASIEV

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A great deal of work has recently been done in the USSR with the purpose of detected rubber plants represented in our flora. The progress made is considerable and a number of valuable rubber plants, among them Kok-saghyz, Tau-saghyz and Krim-saghyz have been discovered. The work has already passed from the stage of exploration to a detailed study of these rubberplants and to their introduction into practice. Yet, botanical exploration cannot be said to have exhausted its possibilities in this direction. The great variety of the physico-geographical conditions in the USSR, the abundance of her flora, as yet not fully known, and still giving room to the discovery of new species and new habitations of species already known, stimulate further floristic researches to be made in a number of other regions. It is probable that such discoveries will be made among the flora of wild growing rubber plants. How great these possibilities are is shown by the history of the thoroughly investigated rubber plant *Scorzonera Tau-sagyz* Lipsch. et Bosse.

After its discovery in 1929 by S. S. Zaretski in the Karatau Mountains, Tau-saghyz attracted exceptional interest as the first productive rubber plant of our flora. During a number of years the attention of many workers was concentrated on this plant and numerous papers reported the results of their research, especially the Journal «Soviet Cautchouc» and other publications of the Rubber and Caoutchouc Institute (VNIK and G.).

After the field work of a series of expeditions organized by the All-Union Institute for Rubber and Caoutchouc Research the distribution of Tau-sagyz was considered to be strictly confined to the (Syr-Darya) Karatau Mounts, as this plant had not been found in any other region inspite of careful search [Makagon(1), A. Zaiceva and I. Belčikova(2)]. Within the habitat in the Karatau Mounts, the brakes of Tau-saghyz, scattered in patches often at great distances from each other, cover altogether a small area, estimated to day as 3500 ha.

Many authors—Pavlov(3), Lipschitz(4), A. Zaiceva and I. Belčikova(2) and others—went even so far as to maintain that Tau-saghyz was a characteristic endemic relict in the flora of Karatau why it could naturally not be found anywhere else. Due partly to this opinion further search for Tau-saghyz in other regions was given up and no crops besides the Karatau Tau-saghyz were introduced into practice.

In 1937 I was charged by the Tajikistan Branch of the Academy of Sciences of the USSR with the geobotanic exploration of the northern districts of Tajikistan. In the course of my work, in autumn I visited the neigbouring region of Kirghisia, and happened to discover small brakes of Tau-saghyz near the head waters of the Isfara river..

The discovery of Tau-saghyz on the northern slope of the Turkestan mountain range at far distances from its already known habitat in the Karatau Mounts, puts an end to the erroneous conception of its narrow endemism and furnishes new material for introduction and selection work. As this discovery presents a considerable scientific and economic interest the actually available data are presented in the present paper.

Tau-saghyz has been found on the northern slope of the Turkestan mountain range at the upper reach of Isfara river. It occurs on the northern slope of the Jiptyr river valley a few kilometres downstream from the mouth of the Tamynghen river. Tau-saghyz is growing here quite close to the norther limit of the wormwood steppes and juniperata, at the elevation of 1900—2100 m, being confined to the stepp (30—60°) slopes of an ancient high terrace with a northern exposition, considerably washed out and intersected with numerous gullies of short streamlets. The soil cover of the slopes is very poorly developed as the fine earth is intensely washed away owing to the steepness of the slopes and the readiness with which the deposits forming the terrace are destroyed. The light coloured soil, particularly that of the upper horizons, contains many stones and pebble. Areas with a developed soil cover every now and then are interrupted by small precipices tali and outcrops of the underlying rocks.

The vegetation cover of the slopes is very thin and greatly varies as to its specific composition and the density of the grass stand, in dependence on the slopes being more or less steep and stony. The degree of covering usually fluctuates between 5 and 25 per cent. On rubbly areas of the slopes together with Tau-saghyz the following plants are usually met with: *Helianthemum songoricum* Schrenk., *Stipa caucasica* Schmalh., *Stipa* sp. nov., *Eurotica ceratoides* (L.) C. A. M., *Artemisia maritima* L., *Silene brachuica* Boiss., *Salsola arbusculaeformis* Drob., *Schrenkia* sp., *Acantholimon alatavicum* Bge. aff., *Allium oreoprasum* Schrenk., *Allium filifolium* Rgl., *Elymus alaicus* Korsh., *Dianthus tetralepis* Nevski, *Astragalus* sp. and others. Not infrequently shrubs of *Ephedra procera*, Fish et May, *Rosa Ecae* Aitch. and *Lonicera* sp. occur as well as single juniper trees (*Juinperus seravschanica* Kom.). On the lower part of the slopes adjoining the rivers *Convolvulus fruticosus* Pall., *Kochia prostrata* (L.) Schrad., *Orysopsis songarica* (Trin. et Rupr.) B. Fedtsch., *Lasiogrostis caragana* Trin. et Runth., *Acantholimon* sp. are common. On slopes exhibiting the character of tali together with Tau-saghyz were observed: *Vicia kokanica* Rgl. et Schmalh., *Asparagus neglectus* K. et K., *Linaria kokanica* Rgl., *Scutellaria orientalis* L., *Ferula transiliensis* (Rgl. et Herd.) Eng. Korg., *Clematis songarica* Bge., *Allium oreoprasum* Schrenk., *Echinopsnanus* Bge.. *Elymus alaicus* Korsh., *Asperula oppositifolia* Rgl. et Schmalh., *Scropularia* sp.

When the material concerning Tau-saghyz collected on the Turkestan mountain range had been worked up it became clear that we had to deal with a new species which could be referred neither to *Scorzonera tau-saghyz* Lipsch. et Bosse nor to any of the four species of Tau-saghyz, referred by M. Kultassov to Karatau. As to its morphological structure the new species is most closely related to *Sc. tau-saghyz* Lipsch. et Bosse from which it differs in the shape of the outer leaflets of the involucre and in some other characters. In the shape of the outer leaflets of the involucre it approaches *Scorzonera longipes* M. Kult., but is well distinct from the latter by the

leaflets being rather thick by a branched flower stalk, a larger achene and other features. A detailed description of the new species of Tau-saghyz from the Turkestan mountain range, which we have named *Scorzonera kirghisorum*, will be found in the next volume of the Bulletin of the Tajikistan Branch of the Academy of Sciences of the USSR.

The rubber-yield of the new species is characterized by the chemical analyses carried out at the chemical laboratory of the Botanical Institute of the Academy of Sciences of the USSR, and amounts in the average to 7.17 per cent. In estimating the given figures it should be taken into account that the material analysed consisted chiefly in strongly suberized and lignified upper parts of old roots, which are known(5) to contain considerably less caouthouc than their other parts.

The content of caoutchouc and resins in separate average samples (Table 1) was determined after Ignatiev's extraction method.

Table 1

Number of samples	Percentage to absolute dry weight of the root	
	Caoutchouc	Resins
1	8.01	4.69
2	7.59	5.25
3	6.69	4.13
4	7.72	4.91
5	5.86	4.70

The determinations of the caoutchouc content (in percentage to the absolute dry weight of the root), after N. B. Kojalowicz's new method worked out at the chemical laboratory of the Institute of Plant Industry are given in Table 2.

Table 2

Number of samples	Caoutchouc p. c.	Number of samplse	Caoutchouc p. c.	Number of samplse	Caoutchouc p. c.	Number of samplse	Caoutchouc p. c.	Number of samplse	Caoutchouc p. c.
1	4.85	11	2.08	21	13.5	31	3.89	41	6.69
2	4.21	12	5.96	22	6.08	32	8.41	42	5.40
3	5.93	13	5.12	23	4.54	33	9.06	43	7.61
4	9.45	14	6.72	24	8.13	34	4.46	44	7.89
5	5.17	15	2.72	25	8.52	35	1.88	45	3.59
6	7.83	16	7.77	26	7.05	36	6.88	46	3.81
7	6.03	17	4.89	27	3.95	37	5.10	47	5.11
8	5.22	18	12.14	28	6.44	38	9.69	48	4.74
9	4.99	19	14.72	29	3.05	39	9.07	49	7.96
10	5.03	20	5.38	30	7.10	40	4.99	50	13.75

The average caoutchouc content determined for 50 samples after Kojalowicz's method, amounts to 6.45 p. c., the maximum reaching 14.72 p. c.

Analyses of the chemical composition of the caoutchouc isolated by means of mechanical methods gave the following figures (in percentage to the absolute dry weight of the root).

1. Pure caoutchouc	66.2
2. Resins	22.78
3. Protein	0.62
4. Adulterations	11.02
5. Ashes	5.02
6. Moisture	26.23

Viscosity in 1 p. c. solution of caoutchouc in benzole, equalizes to 81.00.

As to the content and quality of the caoutchouc, *Scorzonera kirghisorum* does not appears to be inferior to Tau-saghyz from the Karatau and deserves attention as a new valuable rubber plant. The introduction and selection work carried on with Tau-saghyz, ought to be extended to the newly discovered material. The new plant offers a great interest from the standpoint of agricultural practice since its station considerably differs from that peculiar to the species of the Karatau Tau-saghyz. According to literary data Tau-saghyz is distributed in the Karatau chiefly at altitudes ranging from 700 to 1100 m, descending even to lower levels, while in the Turkestan mountain range its area lies at an altitude of 1900—2100 m, close to the limits of wormwood steppes and juniperata. The occurrence of Tau-saghyz in the Turkestan mountain range moreover indicates that its distribution has a wider range and that it may be in other regions too.

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ZOOLOGY

THE EXTERNAL CANALICULI OF THE LATERAL LINE OF FISHES

By D. K. TRETYAKOV

(Communicated by I. I. Schmalhausen, Member of the Academy, 24. I. 1938)

The trunk canal of the seismosensory system which runs under the scales of the lateral line in teleost fishes, manifoldly communicates with the outward medium by means of short canaliculi and their pores. The main canal passes through the plate of the scale in an obliquely-transverse direction to it, while the external canal runs along the plate and this accounts for the difference in the external appearance of the scale of the lateral line from that of the other scales.

The plate of the scale containing the external canaliculus also forms its bony wall. The changes in the form of the canaliculi and of the way in which they communicate with the external medium have up to the present time not attracted the attention of ichthyologists. My observations, made chiefly on fishes of the Black Sea, have disclosed that they show very peculiar relations.

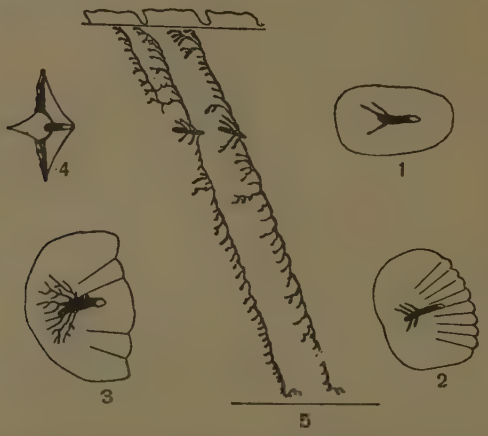
In the majority of teleosts the external canaliculus has the appearance of a short but branching tubule with one external orifice. The latter is just opposite to the internal aperture, through which the main canal of the lateral line enters the next scale. In *Spicaria smaris flexuosa* Raf. there is no external canaliculus and the main canal widens considerably under each scale and communicates with the outside by means of two narrow pores. The position of the latter is symmetric in relation to the midline of the scale, one pore being above and the other below the scale. In the garfish (*Strongylura acus* Lac.=*Belone acus euxini* Günth.) on each scale an external canaliculus with a bony wall starts off from the main canal and runs downwards and obliquely-backwards. It reaches the margin of the plate of the scale where it opens by a pore. With its pore facing the side opposite to the direction of the streams that flush the trunk when the fish moves, such a canaliculus, apparently decreases the force of their vibrations which pervade the main canal. Similar lateral canaliculi I have found in the large flat-fish *Bothus maeoticus Pall.*, and in species of the genus *Teuthis*, inhabiting the Indian Ocean and the Pacific.

Coris julis L. has on its typically cycloid scale of the lateral line a short bony main canal which continues on the external side of the scale as a small canaliculus with an external pore.

The main canal in the scales of the lateral line of the sea crucian (*Sargus-Diplodus annularis* L.) diverges into two bony external canaliculi,

each one of them terminating in a pore. In *Julis pavo* (of the Mediterranean) the main bony canal of the scale diverges, backwards from its internal posterior aperture, into three narrow bony canaliculi. They are distinctly seen (Figure, *1*) on the external side of the dried scale. Two of them usually start off from the superior and one from the inferior margin of the posterior internal aperture of the main canal. Sometimes the two superior canaliculi start off as one common little trunk, while in other cases middle canaliculus runs between the marginal ones at equal distances from both of them. The external canaliculi are but little shorter than the main canal of the scale.

Umbrina cirrhosa L. is characterized by the possession of four external canaliculi on the scales of the anterior half of the lateral line of the trunk. In the caudal part of the body, however, the number of external canaliculi



The external canaliculi of teleost fishes, drawn from preparations of scales of the lateral line, injected with colored Indian ink and dried. *1*—Julis, *2*—Chromis, *3*—Upenoides, *4*—Trachurus, *5*—Trigla lineata (Magn.×3).

is always reduced to one. In *Corvina nigra* C. et V. there are not more than two of them in the anterior part of the body, one of the external canaliculi running obliquely upwards while the other extends along the midline of the scale. In the transition to the caudal part the first one only remains while still nearer the fin it is only the second canaliculus that remains.

The number of external canaliculi of *Mullus barbatus ponticus* amounts to five—six. They are short and together with the main canal account for the characteristic relief of the external side of the scale. [This relief has been depicted in fig. 8 in(1).]

My own observations have shown a great variability in the relations and in the distribution of the external canaliculi and in a young *Mullus* they are relatively longer than in an old one. Three canaliculi are usually found above and two below the main canal. On the caudal pedicle there remains only one upper and one lower canaliculus.

In *Upenoides* I recorded still more numerous external canaliculi. They protrude on the posterior part of the scale (Figure, *3*), in so far as the latter is not covered by the neighbouring usual scales. A similar increase in the number of external canaliculi has also been noted in the parrot-fish (*Scarus*).

In *Chromis chromis* the number of external canaliculi on the scales amounts to five (Figure, *2*), but on the caudal part only four or three canaliculi remain above the main canal.

Owing to the peculiar wing-form and the posterior thorn of the scale in *Trachurus trachurus* the external canaliculi are distinguished by their regular and symmetric distribution (Figure, *4*). From the main canal, two arched, thin canaliculi—upper and lower—start off. They run towards the posterior margin of the scale and are there transformed into their broader divisions which go towards the upper and lower angles of the scale, where they open to the outside by pores. Beside the latter, each division has three more pores on the posterior margin of the scale, while the sections of the canaliculus between the pores are somewhat distant from the margin. The most complicated system of external canaliculi I have found in *Trigla lyra* and in *Trigla lincata* L. The streaked appearance of the skin of the trunk in the latter is due to the long external canaliculi (Figure, *5*) which run obliquely across the body and are scarcely covered by scales. The long external canaliculi of *Trigla* come off from the main canal, one of them running upwards, the other, downwards. The upper one extends to the bony crust, which protrudes over the skin and borders the base of the dorsal fins. Beyond the fins, on the caudal pedicle, the canaliculus of one side passes into the canaliculus of the other side.

The lower canaliculus, which is longer, descends to the base of the anal fin, its lower end often bending backwards. On the caudal pedicle the lower canaliculi of both sides also pass into each other. Throughout their course the long canaliculi give off numerous short tubules with pores at their ends and also along their walls. The longest canaliculi and their terminal tubules run through the fibrous layer of the skin. Most part of the tubules are slightly bent. Sometimes the tubules of one canaliculus unites with the neighbouring canaliculus or forms a supplementary canaliculus provided with tubules too. The scale which covers the main canal contains about four short external canaliculi with a bony wall. The skin of the trunk and caudal pedicle is perforated by countless pores.

In *Trigla lyra* the long canaliculi are covered by scales while the terminal tubules with pores protrude from under the scale, without filling the space between the long canaliculi as densely as in *Trigla lincata*.

The above variations in the system of external canaliculi show that the current notion of the rôle of the seismosensory system with its neuromasts is much too schematic and so far gives no proper conception of its manifold function.

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ZOOLOGY

SEISMOSENSORY CANALS IN THE FLYING FISH

By D. K. TRETYAKOV

(Communicated by I. I. Schmalhausen, Member of the Academy, 24. I. 1938)

Since the publication of the experimental work of Hofer and Parker the organs of the lateral line in fishes are believed capable to perceive currents and vibrations of very low tension. Strong currents and considerable vibration may be produced by the movements of the fish itself. In that case there should be some adaptive mechanism in the fish, by means of which strong currents are converted into vibrations of small tension.

As such an adaptive mechanism I consider the elongate external canaliculi by means of which the main seismosensory canals of the head communicate with the outside medium.

Elongation and complication of the external canaliculi has been noted by me in *Trachurus trachurus* and reported at a meeting of the Ukrainian Academy of Sciences in December 1936; a detailed account is now in press in the Trasactions of the Odessa State University. A still more complicated system of external canaliculi I have found in the head of *Caspialosa* and *Engraulis*. But the most striking features of the seismosensory system I expected to find in the flying fish on account of the maximal speed it develops before the flight in the air.

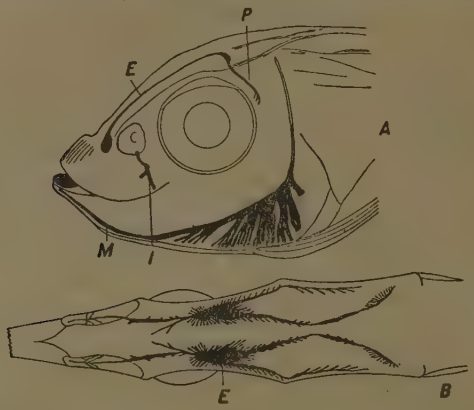
I have injected with colored Indian ink specimens of *Exocoetes (volitans* L.*)* and *Cypselurus* (*exiliens* P. Z. S. Müller) preserved in alcohol and dried for some time in the air. Further drying up of these specimens gives instructive dry preparations with the seismosensory canals injected.

The dorsal bony roof of the skull is traversed from right to left by an unbranched canal (Figure, *A*, *E*). The anterior and the posterior ends of the latter turn away laterally to the margin of the roof. The anterior end of the canal, enclosed in the nasal bone, terminates with a considerable widening anteriorly from the margin of the nasal fossa. This terminal widening is devoid of the bony wall and is covered only by the skin which contains some external pores.

The posterior division of the canal, enclosed in the postfrontal bone, also widens considerably and reaches the postero-superior margin of the orbit. At the point it leaves the frontal and enters the postfrontal bone an oblong external pore is noted. At the margin of the orbit, however, the canal narrows and runs down to the middle of the posterior margin of the orbital canal where it communicates with the outside by two or three pores. There is no canal below the lower margin of the orbit. Below the nasal fossa, however, on the posterior margin of the preorbital bone, a short seismo-

sensory canal runs vertically and communicates with the outside by a pore on the bottom of the nasal fossa itself, or, to be more precise, on its lower wall. From here the canal runs down to the lower margin (Figure, *A*, *I*) of the bone mentioned, protruding on its outer side in the form of a cylinder. In its lower part this preorbital canal widens somewhat, opens by a pore on the anterior side of the widening and is prolonged farther to its lower pore.

The morphological significance of the parts of the seismosensory system of the flying fish becomes clear when compared with that of the pike. Studying after the method mentioned above, the seismosensory canals in the head of the pike, I was able to observe that the supra-orbital canal traverses the nasal and frontal bones in a straight line, without branching and on the supero-posterior margin of the orbit goes over into the postfrontal bone, whence it runs continuously forward under the lower margin of the orbit to the lower margin of the nasal cavity and terminates below the latter.



Seismosensory canals of the flying fish *(A)* and the garfish *(B)*: *E*—supraorbital, *I*—preorbital, *P*—infraorbital, *M*—maxillary.

Along its way the canal has simple pores in its bony wall which are supplemented with likewise simple pores in the skin.

The pike has a supra-orbital and a suborbital seismosensory canal, which unite at the level of the outer margin of the roof of the skull, whence their continuation—the postorbital canal—runs backwards above the margin mentioned to the point of union with the trunk canal. In the flying fish there is no postorbital canal and the canal system of the head is out of connection with the trunk canal. The suborbital canal is, besides, subdivided into a preorbital and a infraorbital canal, only the latter remaining connected with the supra-orbital canal (Figure, *A*, *P*).

Just the opposite to these relations is shown by the supra-orbital canal in the garfish (*Strongylura—Belone acus euxini* Günth.). While also enclosed by the bone of the roof of the skull (Figure, *B*, *E*), it is supplied with numerous, here and there considerably elongated, external canaliculi, which run upwards obliquely towards the surface of the bone to their external pores. In the young garfish the upper wall of the canaliculus protrudes in the form of a cylinder to which the characteristic relief of the roof of the skull is due. The largest number of such canaliculi belong to the middle widened division of the main canal and to its posterior part which extends to the

posttemporal bone. However, there is no connection with the trunk canal in the garfish, just as there is none in the flying fish,.

The middle widened part just mentioned approaches the superior margin of the frontal bone and gives off a descending orbital canal which traverses the latter. Just as in the flying fish, it extends only to the middle of the posterior margin of the orbit, where it opens by two pores. The likeness with the flying fish is also conditioned by the preorbital canal, which is very much like that of the latter.

The maxillary canal in the pike which is separated from the posttemporal one, is in the bony cylinder of the pre-opercular bone prolonged into the dental bone and has only simple pores, by which it communicates with the outside medium. In the flying fish the branching of this canal is very complicated. Descending in the cylinder of the said bone down to the middle of its elevation, the canal gives off downwards and backwards a short external canaliculus (Figure, *A*, *M*) with two pores at the lower end of the latter. In its further oblique onward course the main canal gives off a number of external long canaliculi, some of them closely approaching together while the other dividing into three groups. The anterior canaliculi gradually attain an almost horizontal position. All of them terminate at the lower end of the pre-opercular bone with one or two pores. This margin of the bone is lined by a strip of skin, containing numerous pores arranged in two or three rows.

In its further onward course the main canal enters the dental bone and ceases forming long external canaliculi, but opens to the outside by numerous pores. Thus constructed the maxillary canal communicates neither with the mandibular seismosensory canal nor with that of the trunk.

In the garfish the maxillary canal is not less complicated, than that of the flying fish. Starting, as in the latter, almost at the level of the superior margin of the pre-opercular bone and descending in the cylinder of the latter almost vertically, it at once gives off two long external canaliculi. A third canaliculus goes off from the bend of the main canal and descends to the posterior angle of the pre-opercular bone.

In front of the horizontal bend of the main canal not less than eight external canaliculi are given off and the central ones of them may break up into two branches each. All of them have external pores on the lower margin of the pre-opercular bone.

In the dental bone the maxillary canal extends to the half length of the former. In the part below the eye the canal opens to the outside by simple pores only, but more anteriorly gives off numerous and densely distributed, narrow canaliculi.

It is known that in the pike the lateral line of the trunk appears as it were broken since not in every scale tubuli and pores of external canaliculi are present, while they are in some single scales, situated above and below the lateral line. In the light of my observations this condition is to be explained by the circumstance that from the main canal to these single scales run its elongated external canaliculi. The entire lateral line, however, runs nearer the back.

In the flying fish the lateral line continues along the ventral edge of the trunk and is covered by scales of asymmetric form. The scales are traversed by the unbranched main canal and its short and simple external canaliculi.

In the garfish, on the other hand, the main trunk canal of the lateral line sends off in each scale an elongated canaliculus which descends backwards and downwards to the margin of the scale. Since the scales of the ventral part of the lateral line of the garfish are small, there are very nume-

rous external canaliculi. In many specimens, for the most part, however, on one side only, I find an ascending branch of the seismosensory canals with short external canaliculi. This, apparently, is a remnant of the disappearing connecting canal between the posttemporal and the trunk canals. The ramification of the seismosensory canals facilitates an equable flushing of the body of this fish with the water stream without any considerable tension resulting therefrom. In the garfish the strong currents, resulting from the swift and jerking movements of this fish, by which it is frequently thrown out of the water into the air, are transformed by the elongated external canaliculi into vibrations of small tension. They interfere in the main canals, bringing about an equal stimulation of the neuromasts, distributed only in the main seismosensory canals.

In the flying fish the elongated external canals are found only in the lower pre-opercular part of the maxillary canal. Before rising into the air the flying fish tears along the surface of the water, barely touching it with the lower part of the body. Bearing in mind these movements of the fish it is not difficult to account for the dislocation of the trunk canal to the ventral side and the development of the elongated external canaliculi in the pre-opercular part of the maxillary canal.

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EXPERIMENTAL ZOOLOGY

THE MORPHOGENETIC ACTIVITY OF THE VARIOUS PARTS OF THE HYPOPHYSIS

By A. A. WOITKEWITSCH

(Communicated by I. I. Schmalhausen, Member of the Academy, 27. I. 1938)

In the previous communications (1) concerning the morphogenetic activity of the anterior lobe of the hypophysis, little attention was paid to method. Therefore in the present communication we give a detailed study of the question concerning method.

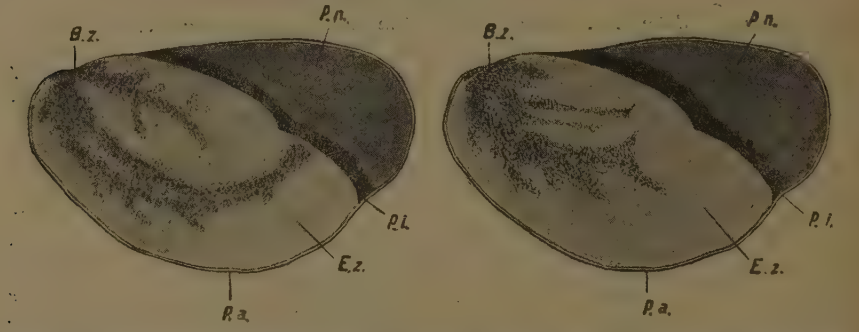
As we have already mentioned, the previous experiments were stimulated by the fact that separate cellular elements are unlikely distributed in the anterior part of the hypophis of bull.

The details of our method of division of these elements of the anterior part consisted in the following.

After extraction and the removal of the surrounding tissues the anterior part was cut with a sharp scalpel along the sagital line. It is absolutely necessary to follow this line of cutting, as otherwise the bazophile zone is extracted with great difficulty and may even be not detected at all. According to this method of dissection the bazophile zone situated in the centre of the anterior part is divided into two approximately equal parts. The region of the bazophile cells is the largest in the frontal part (pars anterior) and gradually decreases towards the posterior lobe. The configuration of this zone greatly varies in separate cases (Figure). In the majority of cases it has the form of a cone with thin branches protruding in all directions into the surrounding tissues of the anterior part. In some cases the basophilic zone penetrates through the whole anterior part in the shape of one or two equal bars. Being sufficiently developed along the sagital section, it is, as a rule, very insignificant on the side sections even closely situated to the sagital line. We came across cases where the bazophilic zone was formed by a small rounded region with thin branches. Finally in some glands the bazophilic zone was totally absent (though we cannot establish the cause of this anomaly). Owing to that these hypophyses were not suitable for our experiments. In the majority of cases the bazophilic zone can be always established macroscopically by its dark color which is due to a strong inflow of blood. If we cut the gland along the border line established macroscopically, the neighbouring regions are always involved. Therefore if the experiment has been undertaken with the aim to test the basophilic zone, it is necessary to cleanse the extracted section, and use for transplanting only the very central parts. Bits of the eozonophilic zone are on the contrary prepared from the

peripheric parts of the gland where the possibility of coming across bazophilic cells is, if not completely eliminated, than reduced to the minimum. At first in working out the methods of investigation we repeatedly undertook a hystological control of the bits of tissues prepared in the way described above. This analysis showed that with all the precautions mentioned above, it is possible to obtain fragments of the bazophilic zone in which the eozinophiles are almost absent. We failed to completely eliminate bazophiles from bits of the eozinophilic zone. They are always present there though in small quantities. In either case it is not necessary to take into account the distribution of the main cells, which as we know are present in the whole tissue of the anterior part.

Of all possible methods of testing the anterior part of fragments obtained as described above, we consider the most rational the implantation of fresh substance. We are bound to come to this conclusion if we take into consideration some of the general viewpoints and the specific traits of the test-object (tadpoles). We gave up the method of injection of the extract, as it includes a moment of specific treatment which may affect the activity of the hormones contained in the gland. Moreover it is difficult to dose the inje-



Scheme of the sagital sections of two hypophysis (cattle) with a differently distributed bazophilic zone. *p. a*—anterior part, *p. i.*—intermediate lobe, *p. p.*—posterior part, *B. z.*—bazophilic zone, *E. z.*—eozenophilic zone.

cted fluid, as owing to the loose condition of the cover part of the injected fluid flows out again. The method of feeding was not suitable since Schmidt(²) experiments performed on rats had shown the absence of the growth stimulation after the introduction of the substance of the hypophysis.

The same negative result was obtained by us in our preliminary experiments on the feeding of tadpoles with the substance of the eozinophilic zone. As to the manner in which or, to be more exact, the locus where the implantation in tadpoles is to be made the only possible way is the intraperitoneal implantation. This has been also confirmed by our transport of the tissues of the hypophysis to birds and mammals. They showed that on subcutaneous implantation the heterotransplanted part is not unfrequently surrounded by connective tissue, cutting off the supply of the hormone to the host. Absolutely safe is only the transplantation of completely fresh tissues. The implantation must be performed immediately after the clearing of a fragment of a definite zone and its rinsing in the Ringer solution or saline. If the fragments to be implanted are kept in the solutions for a considerable period (1—2 hours) their activity is weakened to some extent. As we have observed, the biological activity of the tissue is also lowered if the undamaged anterior part is stored for 24 hours in non-frozen state. Storage especially affects

the eozinophiles. Our communication of the results obtained by means of the method described above was met with the objection that it was impossible to completely separate the two types of the cellular elements of the anterior part which were subjected to differential testing. We have already mentioned that the possibility of an insignificant admixture of foreign elements is not excluded. This occurs, for instance, in the preparation of the fragments of the eozinophilic zone. The most important fact is that this admixture does not affect the results since its biological action is apparently suppressed by the elements of a definite type which are present in overwhelming majority. This is proved by the constant results obtained on the introduction of the substance of a definite zone, namely, the tyreotropic and gonadotropic effect after the implantation of the bazophilic zone, the stimulation of growth with a complete absence of the gonadotropic and tyreotropic effect after the transplanting of the eozinophilic zone. The complete separation of cellular elements is, of course, technically impossible, moreover it is not necessary. We have grounds to assume that its achievement would hardly change the conclusions reached on the basis of the experiments performed with the methods described above.

As it has been established that separate elements of the anterior part with a different structure have an opposite effect depending on whether the basophilic or eozinophilic elements are prevalent in them, this fact must be taken into consideration tn the manufacture of the endocrinic preparations. The peculiarities of the structure of the bull's hypophysis, used as raw material must be taken into consideration in the treatment of the anterior lobe. The latter ought to be first divided with a separate treatment of each part.

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EXPERIMENTAL ZOOLOGY

ON THE QUESTION OF THE BIOLOGICAL ACTION OF A ULTRA-HIGH FREQUENCY ELECTRIC FIELD

By R. J. KOVDA and A. A. WOITKEWITSCH

(Communicated by I. I. Schmalhausen, Member of the Academy, I. 21. 1938)

The present paper concerns the biological action of a high frequency electric field. The experiments were performed on tadpoles of *Rana temporaria*, in which it is possible to change the temperature of the body within a wide range and thus make an attempt to prevent the heating of the animal in the ultra-high frequency field. The chosen object enabled us to observe the possible influence of the ultra-high frequency on the processes of growth and differentiation which are intensely developing during the postembryonal stage of the frog.

The source of energy was provided by a generator with the power of 500 W operating with the valves VK 500. The incandescence voltage of the lamp was 17. 5 V. The volume of the current in the first circuit, 8 A; in the second, 4 A. The potential difference on the anode, 3000 V; wave length, 4 m. The test animals were placed in Petri cup in the field of the condenser. The distance between the plates of the condenser 6 cm; diameter of the plate 17 cm.

Preliminary experiments, undertaken to establish the action of the field under the conditions described above, were performed on warm-blooded (rats) and cold-blooded (axolotles) animals. The rats perished in the ultra-high frequency field after 3—4 minutes, the rectal temperature being 41—42°. The axolotles perished after 3 minutes unless the water gets cool. On artificial cooling of the water the axolotles perish in two hours, the temperature of the water being 17—19° and that of the animal 35°.

The basic experiments were performed on tadpoles which were divided according to their development stage into 3 groups, each consisting of 525 specimens. The tadpoles of the first group belonged to the early stage of prometamorphosis (the bud of the hind extremity was not yet differentiated). The second group consisted of tadpoles whose hind extremities were already divided into sections. The third group consisted of tadpoles in the initial stage of metamorphosis. After a preliminary autopsy of 25 specimens of each group and the establishing of their initial state (weight and linear dimensions of the body and separate organs) they were divided into 5 groups (100 specimens) and exposed to the action of the ultra-high frequency field for various periods of time, viz. 1, 5, 10, 20 and 30 minutes. The temperature of water during experiment did not rise above 18—20° owing to artificial cooling. The tadpoles set for control were kept under analogous conditions, though they were not exposed to the action of the ultra-high frequency field.

Taking into consideration that the action of the ultra-high frequency field may not necessarily be immediate, we were observing the development of the tadpoles for the next 30 days. In order to establish the changes in their development, 10 or 15 tadpoles from each group were dissected every five days.

A brief exposure (1, 5 and 10 minutes) did not affect the tadpoles in their early stage of development. A longer exposure (20 or 30 minutes) partly stimulated the growth of the tadpoles and correspondingly inhibited the approach of resorption processes. The development of the tadpoles in later stages of development, irrespective of exposure, did not differ from that of the control ones. We must point out, however, that the tadpoles exposed for 30 minutes, exhibited a larger weight as compared with the control ones. The results of the additional experiments wiht longer exposures (120—150 min.) enabled us to conclude that the vitality of the tadpoles in the conditions of an ultra-high frequency field is in inverse proportion to their development stage. The further development of the remaining tadpoles (irrespective of the stage) observed till the final stage, presented no deviation from the control.

The results of the experiments made in the course of this investigation lead us to the following general conclusion concerning the influence of ultra-high frequency field on the organism. This is explained by the selective thermal action of the ultra-high frequency field on warm- as well as on coldblooded animals. Although the rate of the reaction of the organism to the influence of an ultra-high frequency field under similar external conditions depends to a great extention on the ability of the animal for thermoregulation. Thus, for instance, the death of warmblooded animals in the ultra-high frequency field takes place after a comparatively short period of time, while the coldblooded animals may exist much longer under conditions of artificial cooling. It should be noted, however, that among coldblooded animals there exist variations due to the age of the animals and the intensity of the heat loss connected therewith. A two hours exposure in a ultra-high frequency field is, as a rule, fatal for axolotles, while tadpoles, under similar conditions, perish in an insignificant number of cases (especially at the early stages of development). The death of warmblooded animals during long exposures is also caused by overheating, in spite of the low temperature of the surrounding medium. Our conclusion as to the selective thermal action of an ultra-high frequency field observed on the larvae of the amphybiae, is in agreement with the results of the experiments performed by Kowarschik([1]) on fish. The author emphasizes the selective action of the ultra-high frequency field on the organism compared with the medium, which is a feature typical for the ultra-high frequency field.

In conclusion we should state it once more that the results of the biological action of the ultra-high frequency field largely depend on the taxonomic position of the animal, the stage of its development and its size.

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EXPERIMENTAL ZOOLOGY

SEX HORMONE AND CHARACTERS OF PARTIAL MONOMORPHISM IN *PERDIX PERDIX* L.

By B. NOVIKOV

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While the rôle of sex hormones in the development of sexual dimorphism has been studied in details, a similar analysis of the so-called monomorphism was undertaken in but a small number of cases. Among mammals this study has been undertaken only with respect to the reindeer (*Rangifer tarandus*), a species in which both sexes possess antlers (monomorphism of the male type). Using the method of castration, Tandler, Gross and others have shown that the development of antlers in *Rangifer tarandus* does not depend on the gonads, while in dimorphic species the development of this character is connected with the secretion of the testis.

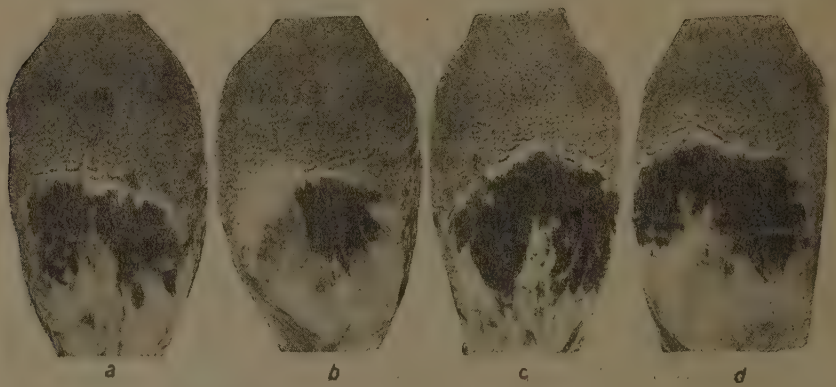
Cardidrois and Ragnier showed on birds that such a character of secondary monomorphism as the bright imaginal disc on the wing of the female is purchased with a heightenning of the threshold of excitation of the featherforming tissue by the ovarial hormone. During the low activity of the ovary taking place in the moulting period, the imaginal disc does develop, but it does not if the feathers are removed during the laying period, i. e. at the moment of a high productivity of the female sexual glands.

The present investigation deals with the question of the mechanism of development of characters of partial monomorphism in the partridge (*Perdix perdix*). Both sexes of this species possess a horse-shoe shaped rusty spot on the lower part of the body. It is always strongly developed in the male, while in the female it shows a great number of individual variations. Some females do not possess it at all, while in others (a very low percentage) it is almost as well developed as in the male. Here we confront a case of partial monomorphism of the male type which must be interpreted as a partial transmission of male characters to the female.

If we take into consideration the fact that in closely related species (genus *Phasianus*) hormonal control takes place in the development of the characters of the sexual dimorphism, we must assume that the realization of partial monomorphism in *Perdix perdix* is connected with a rise of the threshold of response of tissues to the ovarial hormone. It is obvious that in the presence of a comparatively low concentration of the hormone during moulting, in the majority of females conditions are created for the development of male characters. In order to verify this assumption we undertook experiments involving castration, injection of the sex hormone and formation of a new generation of plumage in the period of an increased activity of the gonads. The experiments were performed in the period February—August 1937, on 20 females and 16 males of the partridge. The material was divided into five series: castrated females, normal females, castrated males, normal males, castrated males which received an injection of the ovarial hormone.

The shoe shaped spot on the plumage of the lower part of the body was removed on the right side of the medium line in all birds the left side remaining for control. The new generation of the plumage was examined both macro- and microscopically. The results of the experiment were as follows:

1. In ovariotomized birds which possessed a well developed rusty spot before the experiments no changes were observed in the new generation of the plumage (Figure, *a*). In birds with a weakly developed character



Normal and castrated males and females. *a*—castrated female, *b*—normal female, *c*—castrated male, *d*—normal male.

we observed after the operation a tendency to an increase of the pigmentation zone. Owing to that the symmetry in the development of the spot on the right and left sides was also disturbed.

2. In normal females the plumage of the new generation did not show rusty colours at all, irrespective of how sharp had been the development of the spot (Figure, *b*). The autopsy of these birds showed an active state of the sexual glands and a well developed oviduct (the plucking was performed at the end of March).

3. The castration of the males did not affect the pigmentation of the plumage. In all the operated males the pigmentation of the feathers of the new generation was the same as on the opposite control side of the body (Figure, *c*).

4. The plumage of the normal males the regeneration of which the same as in females was taking place at a period of a high activity of the gonads, showed a tendency to an incomplete development of the rusty pigmentation. The size of the spot on the right «experimental» side was smaller than on the «control» side. As a rule undeveloped pigmentation was observed on feathers near the tail, the colour of which was much like that of the corresponding feathers of the females (Figure, *d*).

5. The injections of folliculine to castrated males (500 M. E. dayly) were begun on the third day after plucking and stopped after the development of the barbs of the regenerated feathers. The rusty pigments had not developed in the feathers of the new generation with the light female colouring.

The results of the experiments performed show that an inhibition of the male characters, transferred to the female, may be achieved by an increase of the activity of the ovary. The assumption that partial monomorphism of the male type taking place in *Perdix perdix* is acquired with a rise of the threshold of the female tissue response to the ovarial hormone, has been confirmed.

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DEVELOPMENT MECHANICS

ON THE DEGENERATION OF SEMINAL TUBULES

By A. M. VASIUTOČKIN

(Communicated by I. I. Schmalhausen, Member of the Academy, 21. XII. 1937)

Phenomena of degeneration of seminal tubules, i. e. of gradual waste of spermatogenous epithelium, have often been noted in literature. However, it is still not clear whether these phenomena may be considered normal or they should be referred to pathological processes.

A. V. Nemilov and I. D. Richter[1] in their recent work (1930) came to the conclusion that seminal tubules degeneration cannot be considered as a pathological phenomenon because of its occurrence in thoroughly normal animals.

First of all they establish that the sertoli's layer of the testis does not consist of separate cells, but presents a syncytial mass including elements of spermatogenous epithelium. The loose of spermatozoa is due to changes in the physico-chemical properties of the layer from the state of gel into that of sol. Consequently, the spermatozoa fall out into the lumen of seminal tubules. This process bears a distinctly expressed seasonal character and intensifies after sexual excitement in animals. Hence A. V. Nemilov[2] makes the conclusion that male sexual hormone is produced by disintegration of spermatogenous cells and sertoli's layer.

Indeed, it is sufficient to examine common school preparations to become sure that the forming spermatozoa lie generally much deeper than it should be expected, according to the current scheme. It is, thus, quite natural that their loosing is not confined to a disintegration of sertoli's cell top and destroys a much greater part of the seminal tubule wall. If we suppose that this process is not strictly limited but extends over neighbouring sections, we have to conclude that there are various stages of seminal tubule degeneration. Thereby, the spermatogenous epithelium undergoes a partial injury, and there is no wonder that immature spermatozoa are occasionally met with in the disintegrating mass.

If the above mentioned authors have established the somewhat seasonal character of the processes of degeneration of seminal tubules in mammals, it is natural to extend the study on animals in which the spermatogenesis has a pronounced seasonal character. To this type belongs our common frog which makes the object of the present investigation.

Let us examine before all the structure of the hibernating frog's spermary. The organ is covered with a rather weakly developed connective tissue capsule, the principal mass of the organ consists of convolute seminal tubu-

les spreading narrower intratesticular discharging tubules (vasa efferentia) lined with low epithelium.

The connective tissue skeleton of this organ is developed weakly; interstitial cells are scarce.

If we pass now to the structure of the wall of the convolute seminal tubules, we may before all establish the presence of large rounded cells, rich in protoplasm, with polymorphous, often segmented nuclei, i. e. spermatogonia. During the winter period they lie in one row, but not in a form of a continuous layer; each spermatogonium appears to be surrounded with peculiar small, so-called follicular cells (La Valette St. George), similar to sertoli's cells in mammals. These follicular cells possess small nuclei, a rather poor protoplasm and, as was indicated as early as in 1890 by Ploetz and 1891 by Bettracchini, are fused together by means of protoplasmic filaments, forming a real net, in the loops of which are settled spermatogonia. The lumen of seminal tubules is thoroughly filled up during the winter period by mature spermatozoa gathered in regular clusters disposed vertically to the wall of the ducts. Generally, under such bundles or spermatozoa heads is situated a more bright nucleus belonging to a cell syncytially connected by its processes with follicular cells. Here we have before us a cell, peculiarly connected with clusters of seminal filaments, called according to Gruenhagen «supporting cell».

The testis of a frog taken by the end of April or at the beginning of May, presents a quite different picture. Here a loosing of lots of seminal filaments and a gradual degeneration of seminal tubules may be observed. Regular clusters of spermatozoa divide into irregular agglomerations lying in the tubules lumens and moving gradually towards the intratesticular discharging tubules. Certainly, the wasting of seminal tubules in amphibians differs from that proceeding in the testis of mammals. Because of the periodic process of spermatogenesis, in mammals not only mature seminal filaments are torn away, but even spermatids and spermatocytes. Such phenomenon is not observed in amphibians. No intermediate forms between spermia and spermatogenia exist in the spermary of a hibernating frog. Therefore, in a spring spermary the bundles of spermatozoa with adjacent supporting cells seem to be cut away from corresponding spermatogenia.

As the spermatozoa are moving away from the tubule the spermatogonia increase in size, multiply and form a continuous row in the wall of the tubule. In June, further multiplication of spermatogonia results in the formation of spermatocytes which are combined in aggregations («spermatocysts») surrounded by follicular cells. In August there appear single spermatozoa, and either in the middle or in the end of September the frog's spermary acquires its final form Of course, not every spermatogonium is performing this transformation—few of them remain laying in their previous form on the walls of tubules, do not change during the whole winter, and become a source of formation of new generations of seminal cells only in the spring.

These data seem to be quite sufficient to demonstrate that the degeneration of seminal tubules is connected with loosing of spermatozoa, this process being expressed in amphibians in its purest form. In the organisms with a more complicated structure of the seminal tubules wall this process has a more complicated character and is followed by a partial destruction of spermatogenous tissue connected with a formation of pathological forms.

Let us consider now whether the authors' inference as to the place of production of the male sexual hormone is correct. We know this may be either interstitial cells or spermatogenous epithelium; but the interstitial

cells in frog's testis are very weakly developed. Then we have to suppose that the spermatogenous epithelium participates in the production of the sexual hormone. The work of Ch. Champy (1922)([3]) confirms this supposition. It establishes a distinctly expressed correlation between the appearance of callosities and the development of spermatozoa. In *Rana exulenta* which is distinguished from *Rana temporaria* by the spermatogenesis proceeding all the year round, the callosities do not develop in absence of mature spermatozoa in the spermary.

Earlier works on the subject are worth remembering too. Tarkhanov (1887) came to the conclusion that frogs copulate only after the filling up of seminal vesicles with spermia and that extirpation of the vesicles leads soon to a separation of the couples.

Steinach (1894), on the contrary, could not find any confirmation of Tarkhanov's conclusions. Dissecting copulating frogs and examining the contents of seminal vesicles he found that copulation precedes the appearance of spermatozoa in the vesicles. This conclusion is supported also by E. Gaupp([5]) (1896), yet it is to be questioned: why copulation takes place in spring whilst spermatozoa appear to have matured already in the fall of the preceding year. So it is, but the copulation occurs inevitably as soon as the degeneration of seminal tubules (i. e. loosing of spermatozoa) sets in and I am inclined to see herein a confirmation of A. V. Nemilow's conclusion as to the place of formation of the male sexual hormone. Very likely, that the same products of partial disintegration of spermatozoa activate the following generation of spermatogonia in which the process of fission sets in energetically.

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DEVELOPMENT MECHANICS

REGENERATION PHENOMENA IN FROG INTESTINES

By A. M. VASIUTOČKIN

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Organs and tissues are considered, indeed, by modern histology as labile dynamic systems developing under the influence of a series of factors, and changing in the course of life according to physiological conditions. But the most striking example of this dynamics («structural dynamics») is offered by metamorphosis phenomenon during which the process of decay of dead elements and that of formation of new viable ones, building up the old yet practically new organ, occur in the latter simultaneously and with maximum plainness.

While metamorphosis phenomena have been studied by histologists in detail, no attention has unfortunately been paid to the question of seasonal changes in organs of amphibia. We certainly have a clear idea of the character of structural changes occuring in liver-cells, genital glands, suprarenal and the thymus gland, but seasonal changes may, undoubtedly, supply us a more abundant material founded on facts, than that we actually dispose of.

It is quite natural that the long period of winter starvation is accompanied by a series of atrophic processes in different organs of the frog. First of all we find in the intestine well pronounced decrease of plication of the mucous membrane. These changes which are visible in the intestine even under a weak magnification, consist before all in shifting of epithelial cells and in their subsequent disintegration which break the regular character of monolayer prismatic epithelium.

As a typical feature one may point out that during the period of starvation the nuclei of some cells are subjected to a sharp hypertrophy, without manifesting any degenerative changes. We have here a phenomenon similar to that described by Drzewina (1912) for fish intestines. The gradual decay of epithelial cells results in a formation in the epithelium of vacuoles filled up with small lumps of chromatine, grains of pigments, and other products of disintegration of cell elements which undergo further gradual dissolution. This vacuolation of epithelium leads to a still greater sponginess of the epithelial layer and gives thus a picture similar to what may be observed in the intestines of a frog-tadpole during metamorphosis.

As according to several authors the regeneration of intestinal epithelium during metamorphosis proceeds at the expense of special substituting cells [Reuter(1), 1900; Duesberg(2), 1906; Krehmer(3), 1927, etc.], we have to consider the regeneration of intestinal epithelium in spring time as an ana-

logous process. The substituting cells seem to form from ordinary prismatic cells by way of differentiation, their activity being enhanced by the disintegration products of epithelial elements. The intensive multiplication of these substituting cells results in a formation of «special cellular nidi», a symplastic mass reminding of rudiments of glands.

A very interesting picture is observed when phenomena of decay and formation occur in the villi intestinales simultaneously. We see a fused protoplasmic mass with scattered degenerating pycnotic nuclei, and just under it, a symplastic mass formed by substituting cells which show up owing to their «embryonic character». While the old epithelium is dissolving these nidi of cambial elements become a source of formation of new epithelial cells.

Thus in the regeneration of intestinal epithelium during spring periods we have processes which remind us of regeneration phenomena observed in the case of metamorphosis, that is we meet here with a regeneration of the so-called embryonal type.

The following phenomenon is worthy mentioning too. During the metamorphosis of *Alytes obstetricans* there was observed a transformation of these symplastic mass-concentrations into peculiar «epithelial cysts» with a central cavity and nuclei disposed like radii. According to K. Reuter, these cysts are sources of new epithelial cells. Similar pictures could occasionally be observed on my preparations. I suppose, however, that the fact of formation of cellular nidi, reminding of of glands, enables some conclusion, disregarding whether they are transformed into epithelial cysts or not.

It is known that Lieberkühn crupts are absent in frog intestines, whereas in tailed amphibians they are represented by rounded accumulations of cell which received the name of «Epithelsprossen» and «bourgeons germinatifs». I believe that the formation of cellular nidi in regeneration both during metamorphosis and at spring periods is a peculiar manifestation of histological atavism, an example of «atavistic manifestations accompanying the regeneration» (V. Šimkevič, 1923), when the organ does not regenerate in the form it had in this individual, but is shaped as in the ancestors ot the given animal form.

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